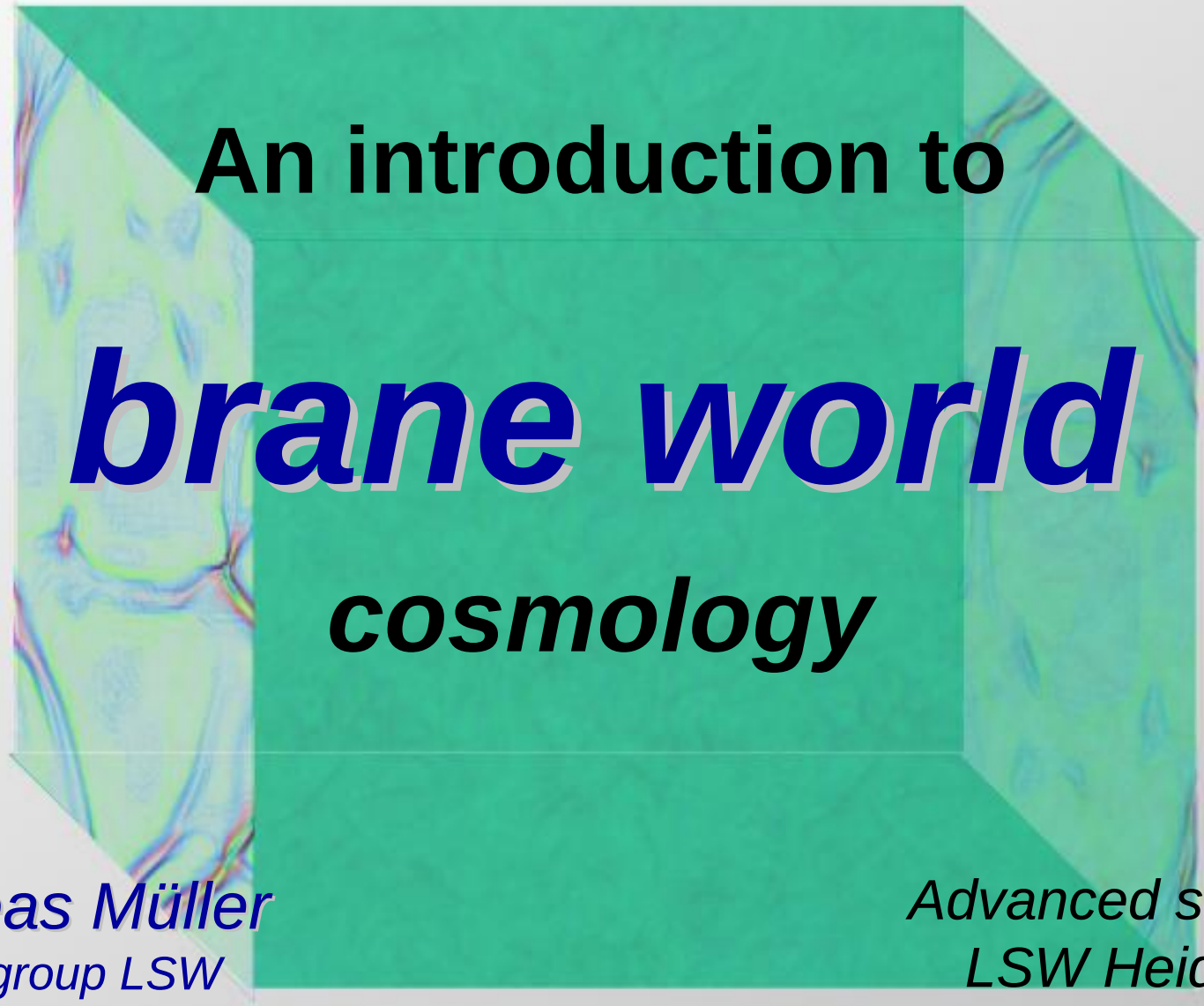

An introduction to

brane world

cosmology

Andreas Müller
Theory group LSW

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http://www.lsw.uni-heidelberg.de/users/amueller

Overview

- principles
- bulk and brane
- extradimensions
- compactification
- ADD vs. Randall-Sundrum
- scalar fields
- brane collisions
- ekpyrosis and cyclic universe

Standard cosmology

- GR world is 4D manifold: space-time
- Robertson-Walker metric
- Λ cosmology
- cosmological constant, dark energy
- ΛCDM in a flat, expanding universe
- FRW equations
- Hubble constant
- inflation
- Big Bang

Motivation to brane world

- coincidence problem:
  $\Omega_\Lambda \sim \Omega_m$
  solution: Λ becomes dynamical
  quintessence models (QCDM),
  brane worlds
- hierarchy problem:
  weakness of gravition!
  Planck scale ~ 10^{19} GeV
  electroweak scale ~ 1 TeV
  16 decades discrepancy!
  solution: **extradimensions**,
  brane worlds

Extradimensions and brane worlds

- extradimensions (XDs):
  - ~1920: *Nordström, Kaluza-Klein*
  - ~1990: renaissance in QFT, SUSY; *Antoniadis*
- implications from string theories and M-theory: compactified extradimensions
- count XDs in particle accelerator black holes?
- standard model of particle physics is confined on a hypersurface, the **brane**

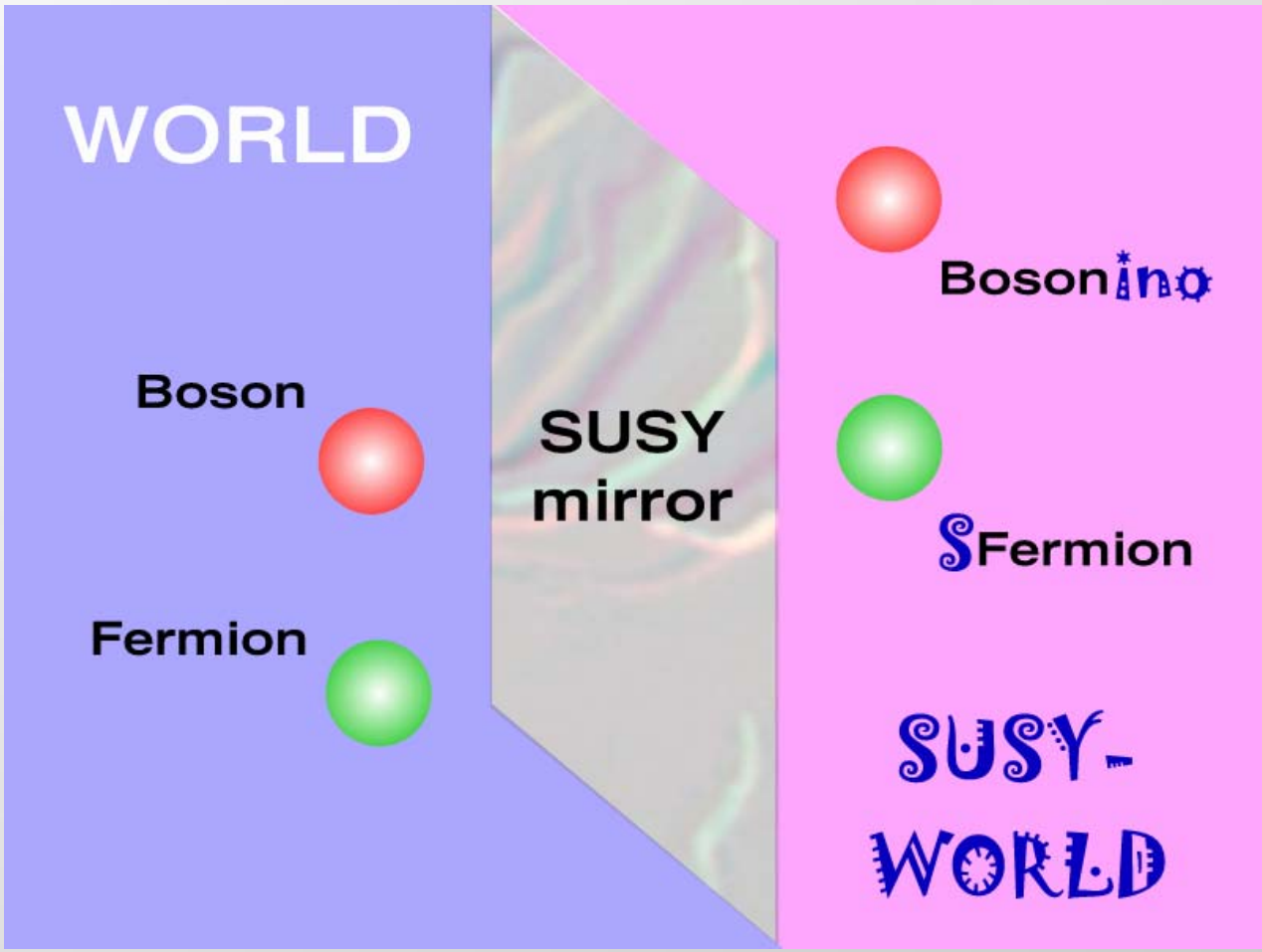
  (etymology from *membrane* by *Paul Townsend: p-brane has dimension p*)
- brane is embedded in higher-dimensional space, the **bulk**

Bulk – brane topology

Brane world zoo

- number of extradimensions
- compactification vs.
  non- compactification
- flat vs. warped bulk geometry
- number of branes
- static vs. dynamical branes
  (brane collisions)
- vacuum bulk vs. bulk scalar fields

Supersymmetry



SUSY mirror creates particle zoo

String theory

- 5 supersymmetric string theories connected via **dualities** hint for **M-theory**
- 11D supergravity (SUGRA) connects GR with SUSY
- SUGRA is low-energy limes ($l >> l_{Pl}$) of M-theory and therefore all string theories
- 11D SUGRA has 11th dimension compactified on an orbifold (with Z_2 symmetry)
- boundaries of 11D space-time are 10D „planes“
- on planes E_8 gauge groups confined
- **Calabi-Yau threefold** represents compactified space of 6 dimensions of 11D („microscopic ball“)
- **heterotic string theory** $E_8 \times E_8$ results in brane world (*Horava & Witten 1996*)

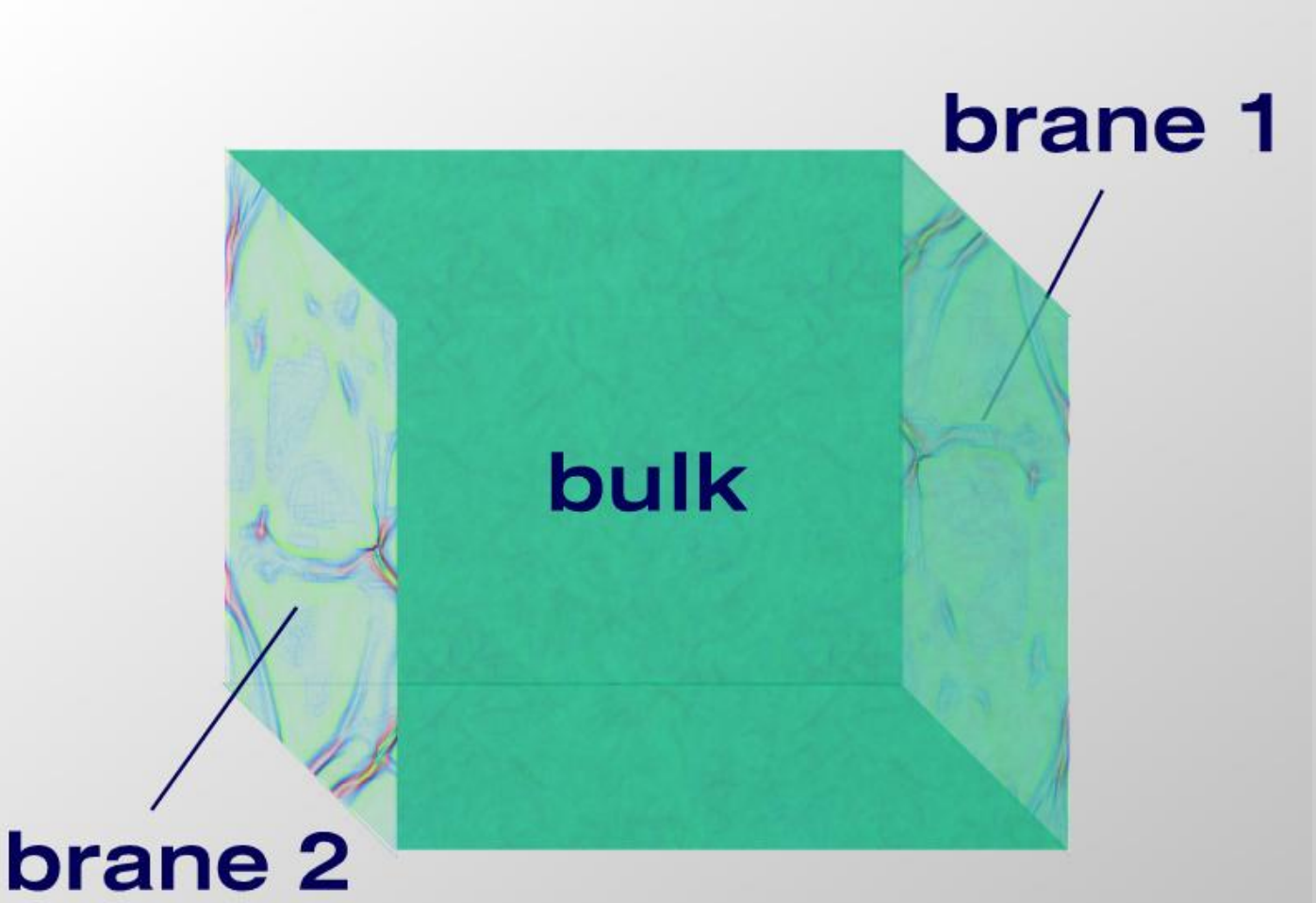
String theory: ADD model

- motivation for 5D space-times
  with 4D boundary branes
- ADD scenario: **large extradimensions (LXDs)**
- **flat** bulk geometry 4+d
- d compactified extradimensions
- reduced Planck scale:
  $M^2_{P,ADD} = M^{2+d}_{fund} R^d$
  M_{fund}: 4+d Planck scale
- radii < R: **non-Newtonian gravity**

Newton‘s law modified

- SM restricted to brane, gravity propagates into bulk
- extradimensions compactified to radius R
- 1st implication: **Newton $1/r^2$ injured** for radii ~ R
- tests with Cavendish experiments show **no evidence** up to now
- if LXD exist, then R << 1 mm

2-brane system



hypersurface: $\mathbf{D}_{brane} = \mathbf{D}_{bulk} - \mathbf{1}$

Randall-Sundrum I model

- **2-brane system**
- **warped** (*curved*) bulk geometry 4+d
- bulk metric is slice of **Anti de Sitter** (AdS_5) space-time, $\Lambda < 0$, 5D:

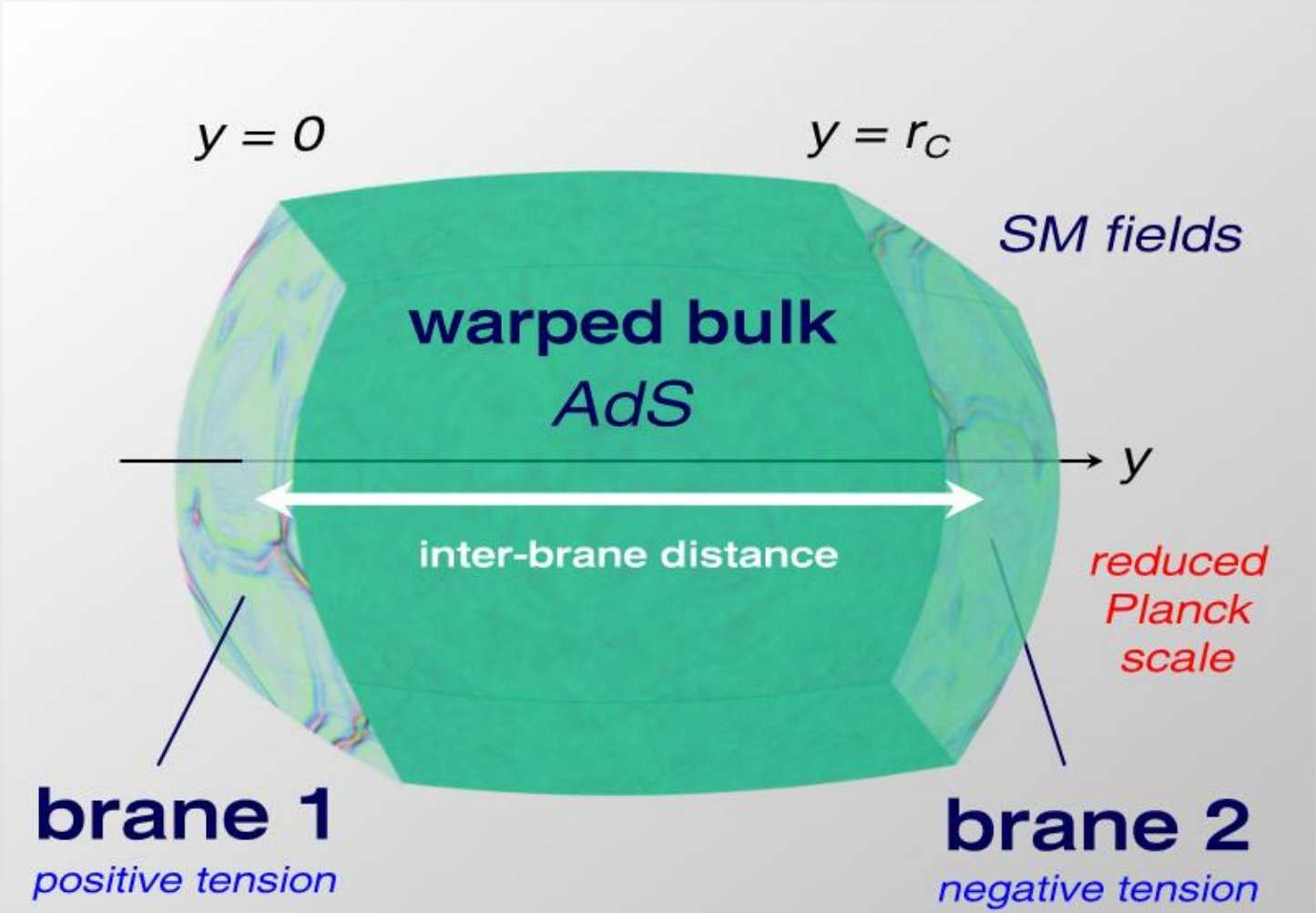
$$ds^2 = e^{-2K(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

- <u>new:</u> restauration of Newton‘s law on brane with positive tension embedded in infinite LXD!
- solution of the **hierarchy problem**

(10^{19} GeV Planck vs. 100 GeV electroweak):

2-brane model (**RSI**)

Randall-Sundrum I model



remark: branes are **Minkowski-flat**

Randall-Sundrum I model

- highly-curved AdS background
  - ➢ implies **large gravitational redshift** of energy-scale between branes
- hierarchy due to large **inter-brane distance** r_c
- Planck scale (on negative tension brane) is reduced to TeV:

  $M^2_{P,RS} \sim \exp(2kr_c)\, M_5^3/k,\ k = (-\Lambda_5 \kappa^2_5/6)^{1/2}$

  Λ_5: 5D negative cosmological constant on bulk

  κ_5: 5D gravitational coupling constant

  M_5 : 5D Planck mass
- **fine tuning problem:**

  radius of LXD, r_c, tunes hierarchy scale
- **radion** as bulk scalar field (later!)

Randall-Sundrum II model

- AdS background
- send negative tension brane to infinity
- effectively **non-compact** 1-brane model
- contrast to KK (all XDs compactified):

  gravitational field has **continuum** of KK modes
- consequence:

  correction of gravitational force on brane

Randall-Sundrum II model

- modified Newton potential for point masses on the brane

$$V(r) = \frac{G_N m_1 m_2}{r}\left(1 + \frac{l^2}{r^2} + O(r^{-3})\right)$$

with $l^2 = -6/(\Lambda_5 \kappa^2_5)$

- experiments prove $l < 1$ mm

Randall-Sundrum II model

- modified Friedmann equation in 5D

$$H^2 = \frac{8\pi G}{3}\rho_M\left[1+\frac{\rho_M}{2\sigma}\right]+\frac{\Lambda_4}{3}+\frac{\mu}{a^4}$$

$$\frac{8\pi G}{3} = \frac{\sigma}{18}$$

$$\frac{\Lambda_4}{3} = \frac{\sigma^2}{36}+\frac{\Lambda_5}{6}$$

split in matter and brane tension

- tuning between Λ_5 and σ establishes $\Lambda_4 = 0$
- gravitational constant depends on tension σ
- μ is *dark radiation* term

Observational constraints

- **nucleosynthesis**

  $\sigma > (1\ \mathrm{MeV})^4$,

  then classical Friedmann eq. established at z_{nucl}, otherwise abundances significantly changed

- **Newton‘s law tests**

  $\sigma > (100\ \mathrm{GeV})^4$, $\kappa_5^{-3} > 10^5$ TeV,

  then classical Friedmann eq. established at z_{nucl}, otherwise abundances significantly changed

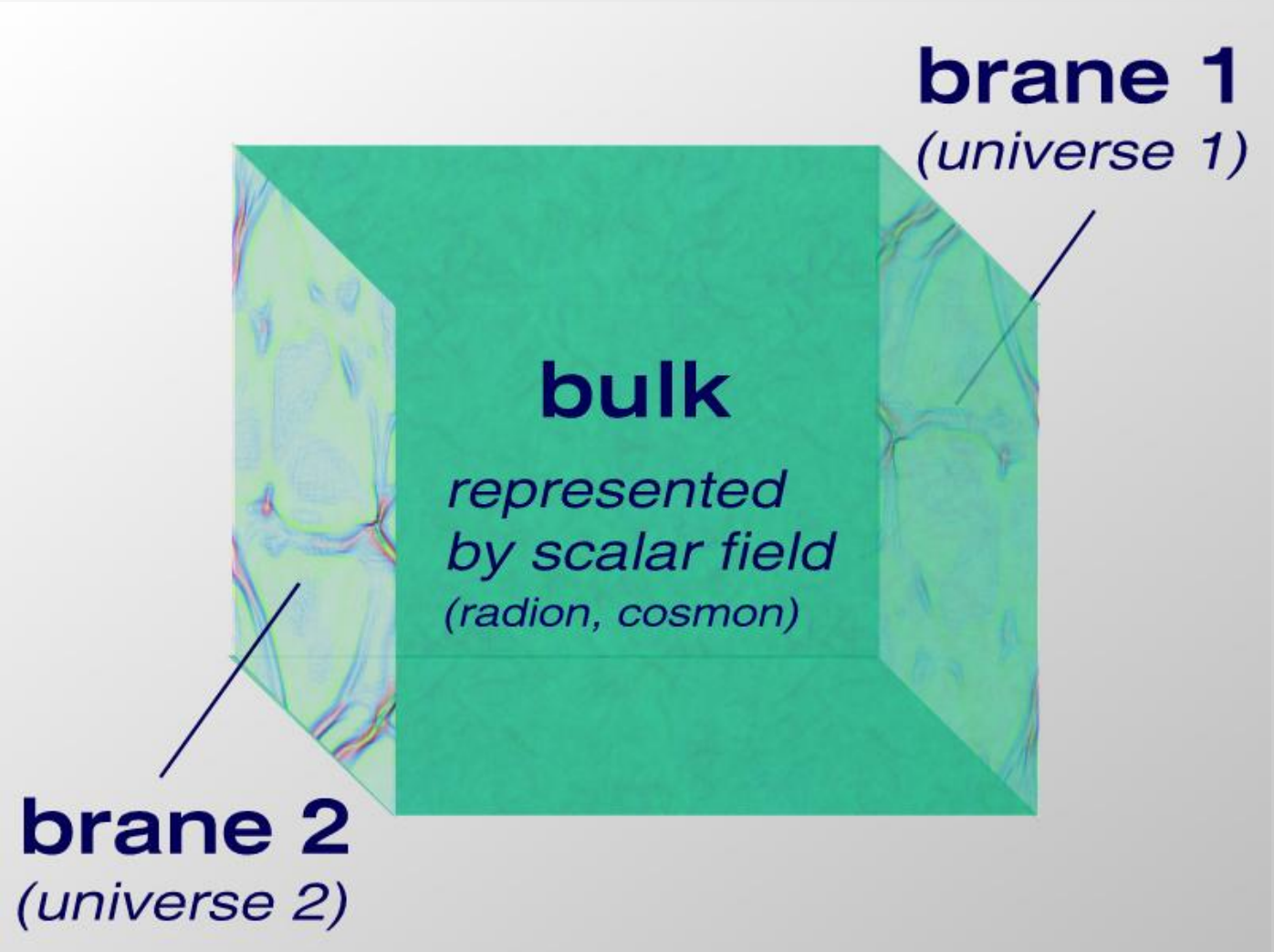
- **cosmology**

  $\mu < 0.1\ \rho_{phot}$; typically assumed $\mu = 0$

Technical aspects

- start with **action** (Einstein-Hilbert, ansatz for brane: contains tension)
- derive **Einstein equations** as EOM, including **Klein-Gordon equation**
- **solve** this set of equations (integration...)
- deduce **bulk metric** (AdS, Schwarzschild etc.)
- identify **tunings** ($\Lambda_5 - \sigma$ – relation etc.)
- discuss resulting **cosmology**, e.g. modified Friedmann equations, effective cosmological constants...

Bulk scalar field



Bulk scalar field

- up2now: *empty* bulks
- now: fill bulk with **scalar field**
- **dynamical** brane configurations!
- bulk back reaction parametrized by *Weyl tensor* and *loss parameter*
- discuss modified Friedmann eq.
- Klein-Gordon eq.:

  time dependence of scalar field
  - trace of energy-stress tensor on brane
  - gradient of bulk potential
- G becomes **time-dependent**: G = G(z)
- fine-structure constant has time evolution
- bulk scalar field can play role of **quintessence**

Scalar field

- energy density, pressure, potential energy

$$\rho_\phi = \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi)$$

$$p_\phi = \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi)$$

e.g. **inflaton**

- full evolution described by:
  - modified Friedmann eq.
  - Klein-Gordon eq.
  - Raychaudhuri eq.
- assume *slow-roll* regime
- result: brane world effects slow-roll scenarios

Scalar field - inflaton

- in slow-roll regime (1):
  high potential vs. low kinetic energy of scalar field
- high negative pressure drives expansion of universe
- fall into potential well (2):
  inflation ends, inflaton field oscillates and decays into matter and radiation

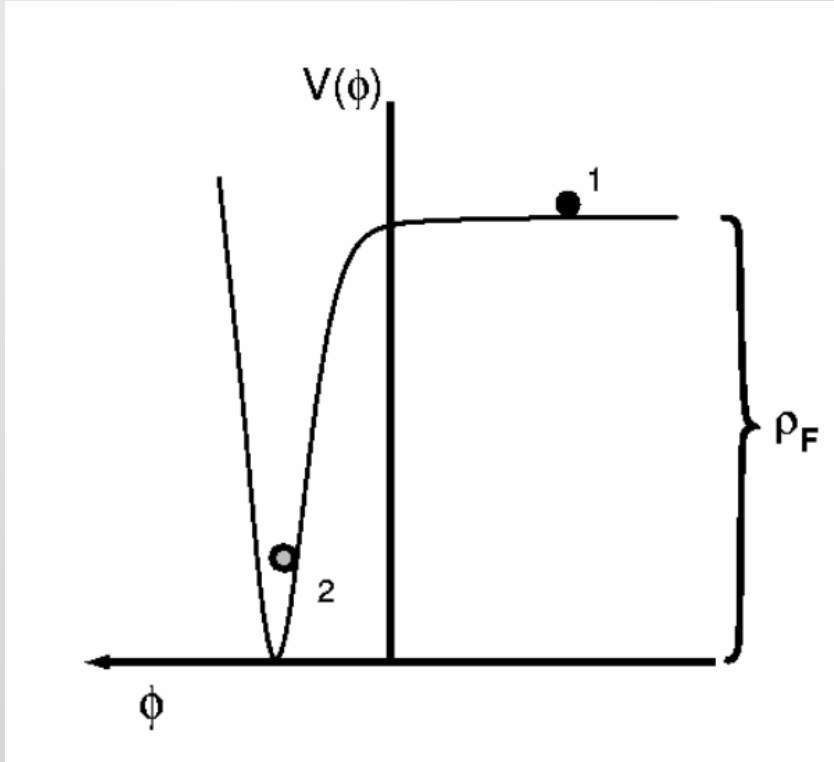


figure: Steinhardt & Turok 2002

$$\rho_\phi = \frac{1}{2}\phi_{,\mu}\phi^{,\mu} + V(\phi)$$

$$p_\phi = \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi)$$

Cosmology of 2-brane systems

- motivation: 1-brane system + scalar field generates **naked singularity** (bulk singularity, AdS horizon). This can be shielded with 2nd brane.
- bulk scalar field fixes inter-brane distance in RSI model
- consider **variable** inter-brane distance
- **radion**: inter-brane distance plays role of scalar field
- small radion field at late times: negative tension brane moves towards bulk singularity and might be destroyed or repelled

Cosmological constant

- observed $\Lambda \sim 0$ invokes extradimension effect
- hierarchy problem reemerges in a fine tuning problem of the inter-brane distance
- self-tuning idea: XD highly curved, but brane stays Minkowski-flat. <u>But:</u> bulk scalar field produces naked singularity. Vanishes with a 2nd brane.
- **Friedmann equations modified** at high energies ($\rho_m >> \sigma$) in brane world models:

$$\boxed{H \sim \rho_m}$$

instead of classical 4D:

$H \sim \rho_m^{1/2}$

Ekpyrotic scenario

- initial state two flat 3-branes: our progenitor universe and a „parallel“ universe
- branes approach as mediated by radion field
- in **brane collision** event kinetic energy is transformed into quarks and leptons
- **no big bang singularity**!
- finite temperature 10^{23} K
- homogeneous and flat universe
- no inflation!
- no magnetic monopole formation (T too small)

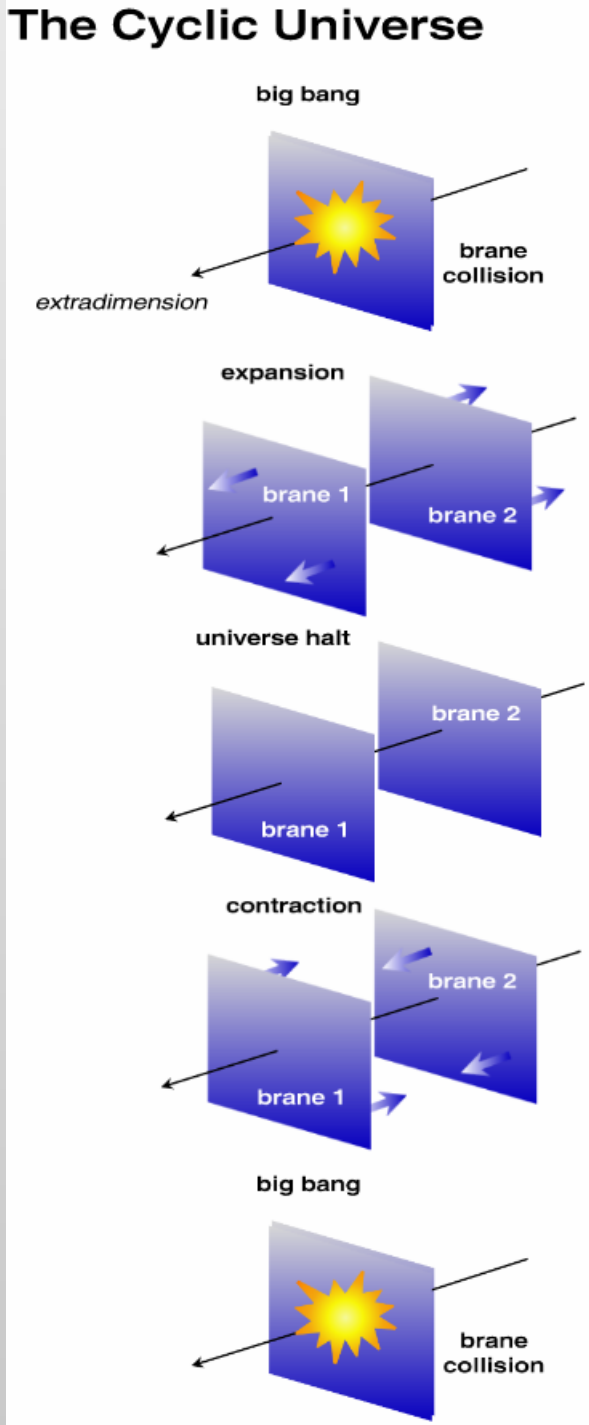
Khoury et al. 2001

Cyclic Universe

- **periodic sequences** of ekpyrosis
- cycle of

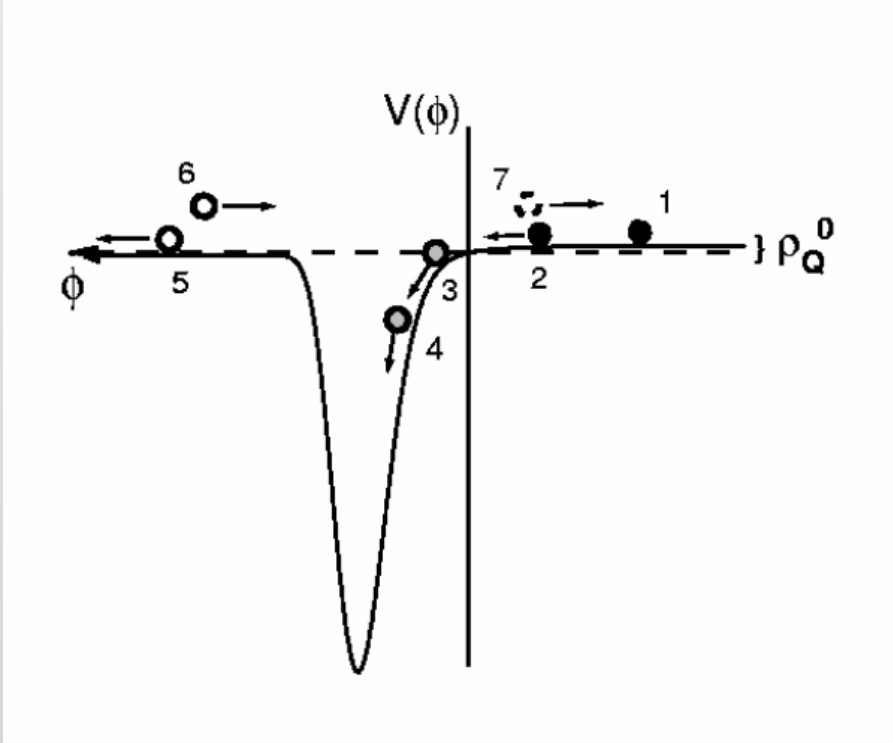
  big bang, expansion, contraction, big crunch
- scalar field acts as dark energy (precisely **quintessence**) that accelerates and decelerates
- scalar field has natural geometrical interpretation in string theory

Steinhardt & Turok 2001



Cyclic Universe

- (1) E_{pot} dominant
- (2) roll to well due to universe expansion and cooling
- (3) $E_{pot} = 0$, E_{kin} dominates universe, expansion decelerates
- (4) $E_{pot} < 0$, contraction
- (5) acceleration out of the minimum, scale factor zero: „crunch“
- (6) reheating of universe from kinetic energy conversion into matter and radiation
- (7) rush back



Steinhardt & Turok 2002

Brane Worlds – συν–οΨις

- existence of extradimensions
- $\Lambda = 0$ on the brane easily managed
- impact of brane cosmology on early universe

  $H \sim \rho_m$ instead of $H \sim \rho_m^{1/2}$
- **dark energy, quintessence** represented by scalar field
- ekpyrosis: 1st **explanation** of big bang!
- universe may evolve in cycles

Open questions

- effects of bulk gravitation on CMB and LSS
- boundary conditions on the brane
- variations of the bulk scalar field around the brane
- bulk scalar field as dark energy constituent
- shielded bulk singularity
- singularity problem in brane collisions

Cosmology news

- **$w = p/\rho = -1$** *Einsteins cosmological constant* Λ

  high-z SN Typ Ia permanence measurements

  (*Riess et al., February 2004*)

- distance ladder

  z ~ 7 lensed IR galaxy

  (*Kneib et al., February 2004*)

  z ~ 10 lensed IR galaxy Abell 1835 IR 1916

  lens magnification factor 25-100, 5 x 10^8 $M_\odot$, ISAAC/VLT

  (*Pello et al., March 2004, astro-ph/0403025*)

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Abbreviations and Acronyms

- ADD: Arkani-Hamed, Dimopoulos & Dvali model
- AdS: Anti de Sitter space-time
- BH. Black Hole
- CMB: Cosmic Microwave Background
- D: Dimension
- EOM: Equation of Motion
- FRW: Friedmann-Robertson-Walker
- GR. General Relativity
- GW: Gravitational Wave
- KGE: Klein-Gordon Equation
- KK: Kaluza-Klein
- ΛCDM: Λ cosmology with cold dark matter
- LSS: Large Scale Structure
- LXD: Large Extra Dimension
- QCDM: quintessence cosmology with cold dark matter
- QFT: Quantum Field Theory
- RSI: Randall-Sundrum model I
- RSII : Randall-Sundrum model II
- SM: Standard Model of Particle Physics
- SUGRA: supergravitation
- SUSY: supersymmetry
- XD: Extra Dimension