

Relativistic Emission Lines of Accreting Black Holes

Andreas Müller

21. Mai 2003

Quasare und früher Kosmos

Forschungsseminar LSW

Overview

- Motivation
- AGN paradigm
- AGN X-ray spectra
- X-ray fluorescence
- Coronal irradiation
- Rotating black holes and Kerr ray tracing
- Plasma kinematics
- Accretion theory and radiation
- Radial drift model: disk truncation
- Simulated disk images: g-factor, emission
- Emission line calculation
- Emissivity models
- Emission lines: calculation, studies, criteria, classification, observation

Note:

All radius declaration were in units of the gravitational radius

$$1 r_g = GM/c^2$$

In general, relativistic units were used $G=M=c=1$

$$1 r_g := 1.0$$

Motivation

In general

- probing strong gravity
- verify or falsify event horizon?

Black Holes vs. Gravastars

(few hope because strong redshift suppresses *any* information)

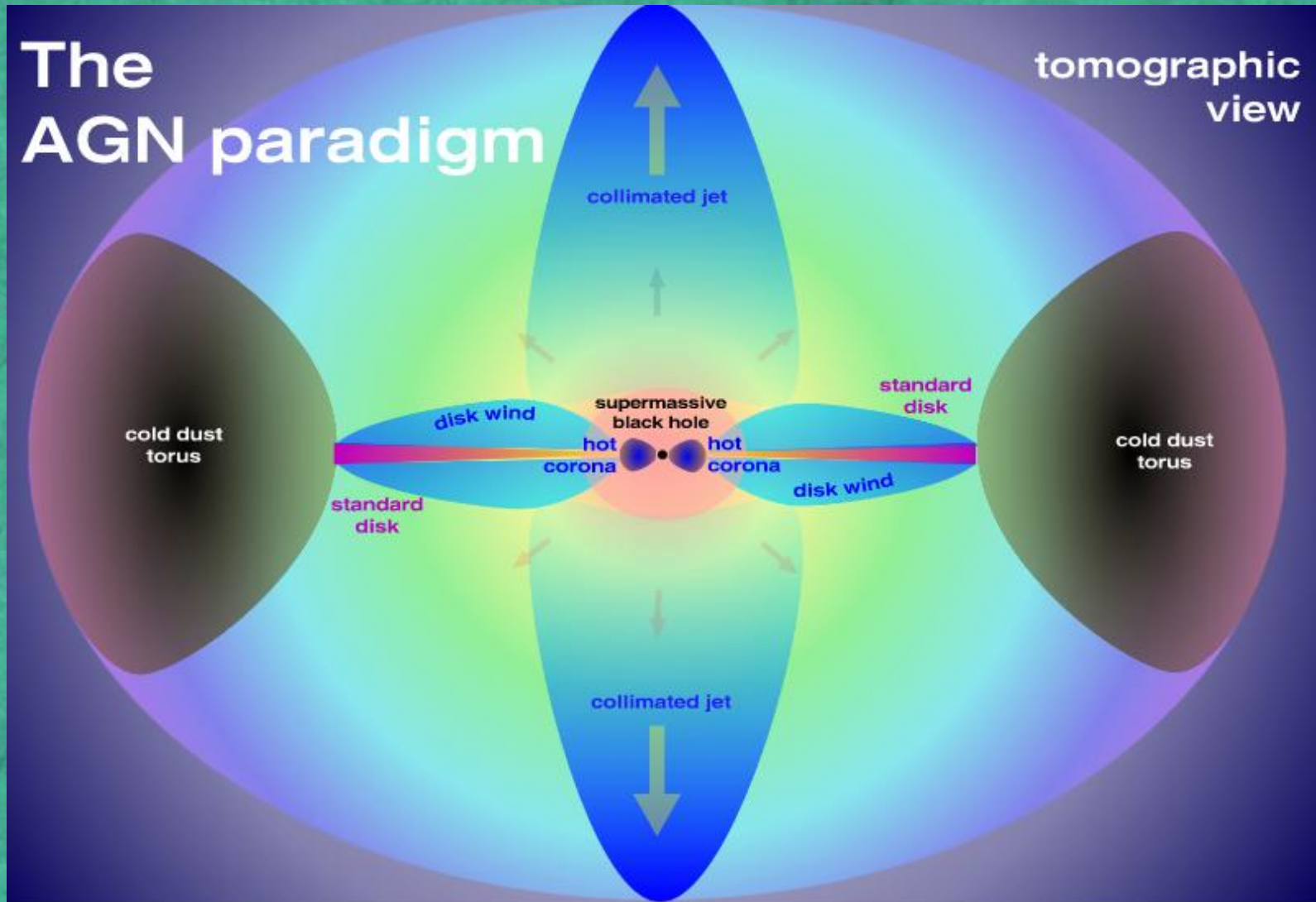
- measure parameters in accreting black hole systems
(AGN, microquasars, globular clusters)

Cosmological

- emission line diagnostics for **Quasars** feasible
- highest redshift today: $z \sim 0.16$ (3C 273)
- extension to **Early Universe** expected

The AGN paradigm

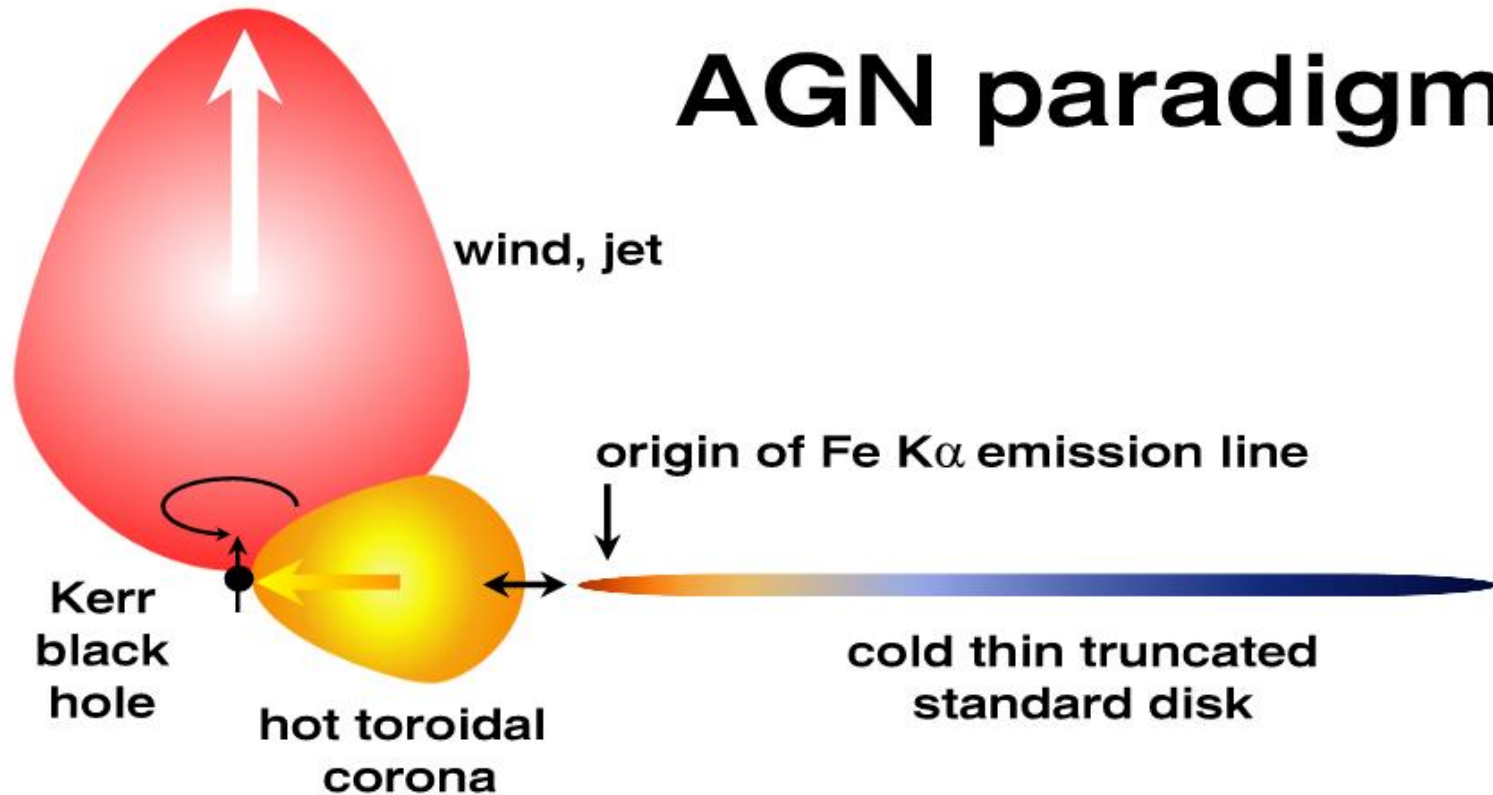
Global topology: kpc-scale



X-ray emitter

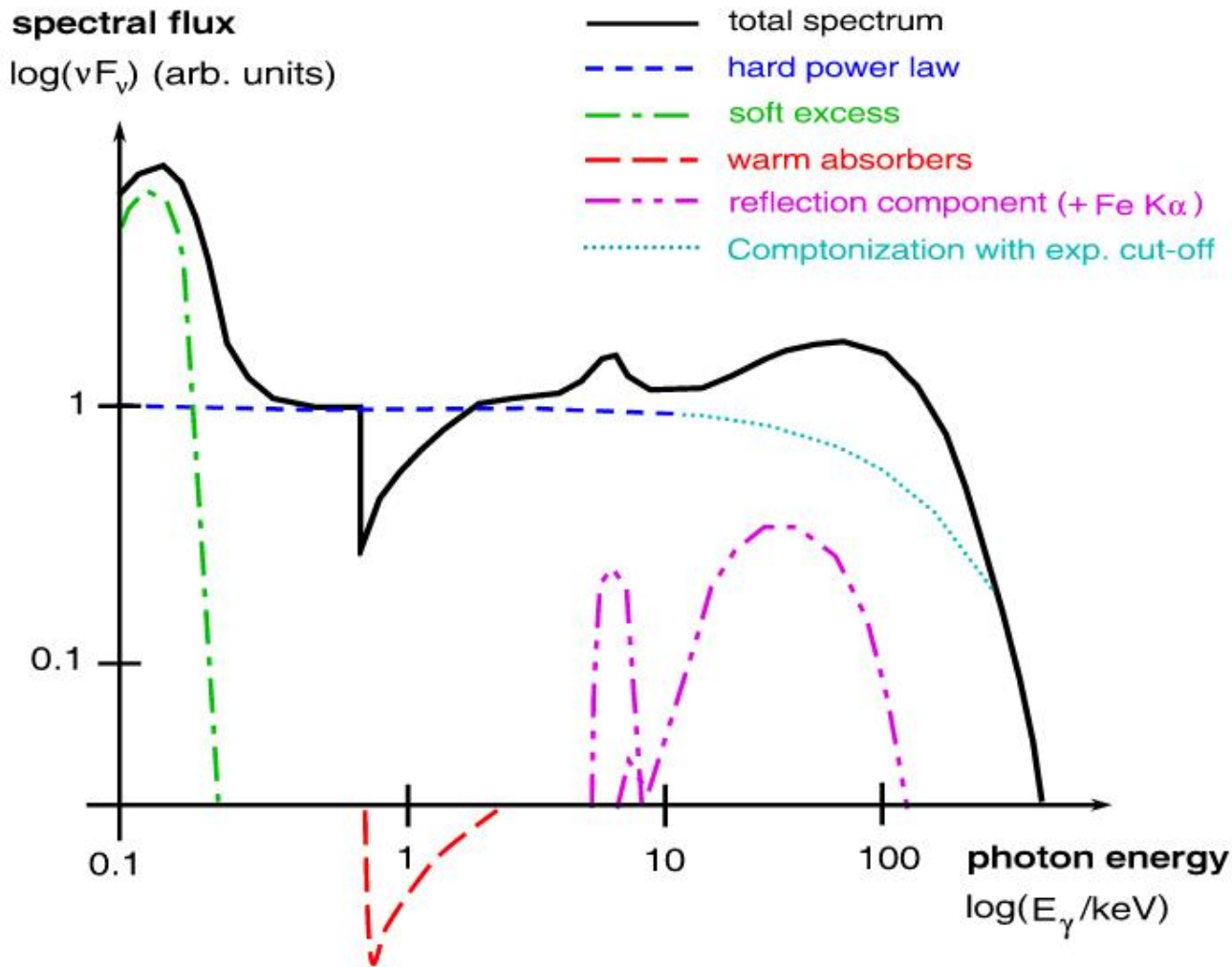
Accreting black holes: pc-scale

AGN paradigm



X-ray AGN spectra

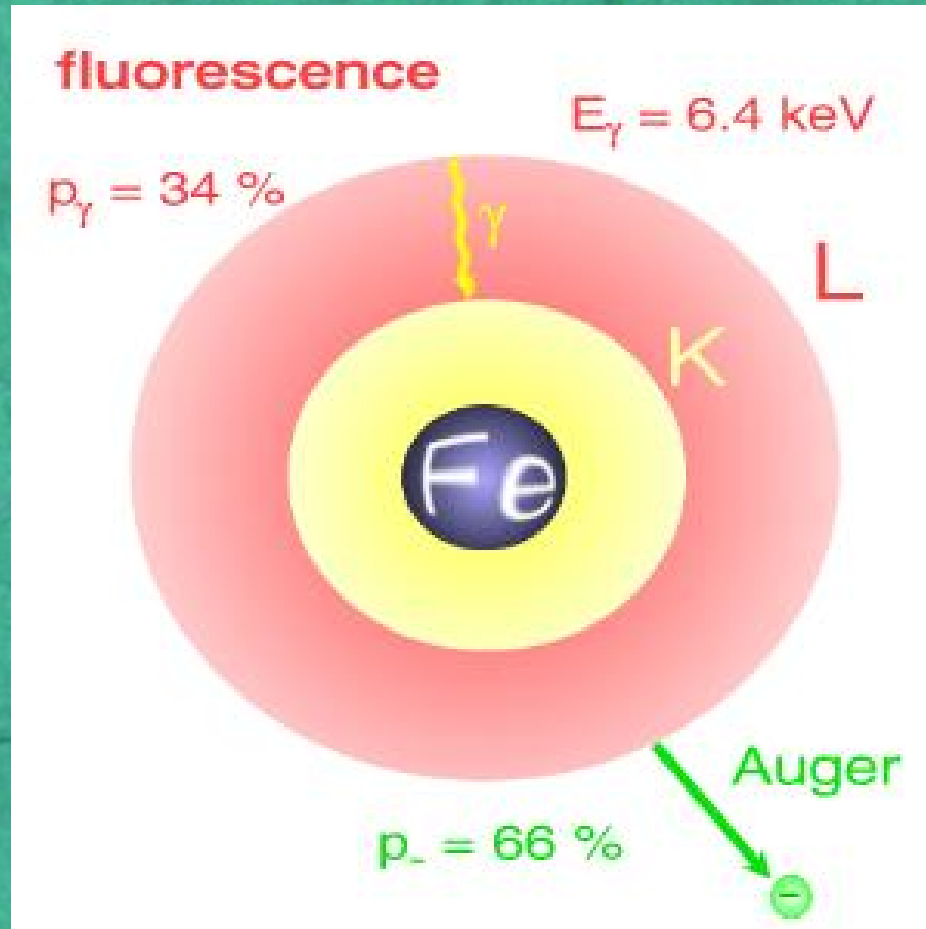
Spectral components



(plot idea by
A. Fabian 1998)

X-ray fluorescence


Fe $K\alpha$



X-ray fluorescence

Prominent species

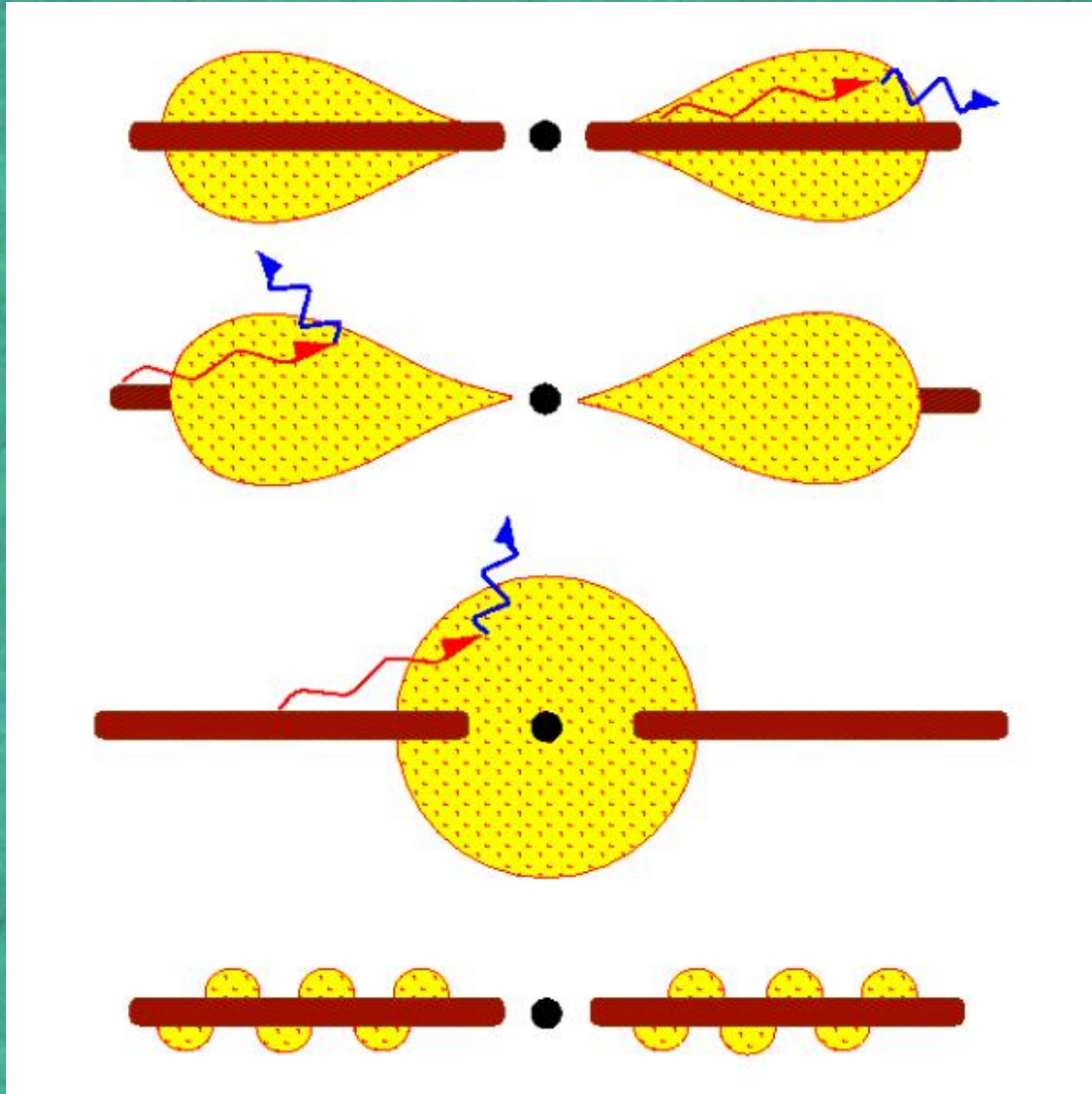
Fe $K\alpha$	6.40	keV
Fe $K\beta$	7.06	keV
Ni $K\alpha$	7.48	keV
Cr $K\alpha$	5.41	keV



decreasing
relative line
strength

*Dependency of these rest frame energies
on ionization state!*

X-ray illumination Corona geometries



slab, sandwich

sphere+disk
geometry

patchy, pill box

(Reynolds & Nowak 2003)

X-ray illumination

The corona problem

- corona geometry and location
still open question!
- models:
 - slab corona (SSD, slim disk)
 - patchy corona
 - sphere+disk geometry (ADAF)
 - on-axis point-source (jet)
- observational technique:
reverberation mapping
- theory: ***radiative GRMHD in 3D***

Rotating Black Holes

Kerr geometry

$$ds^2 = -\alpha^2 dt^2 + \tilde{\omega}^2 (d\Phi - \omega dt)^2 + \rho^2 / \Delta dr^2 + \rho^2 d\Theta^2$$

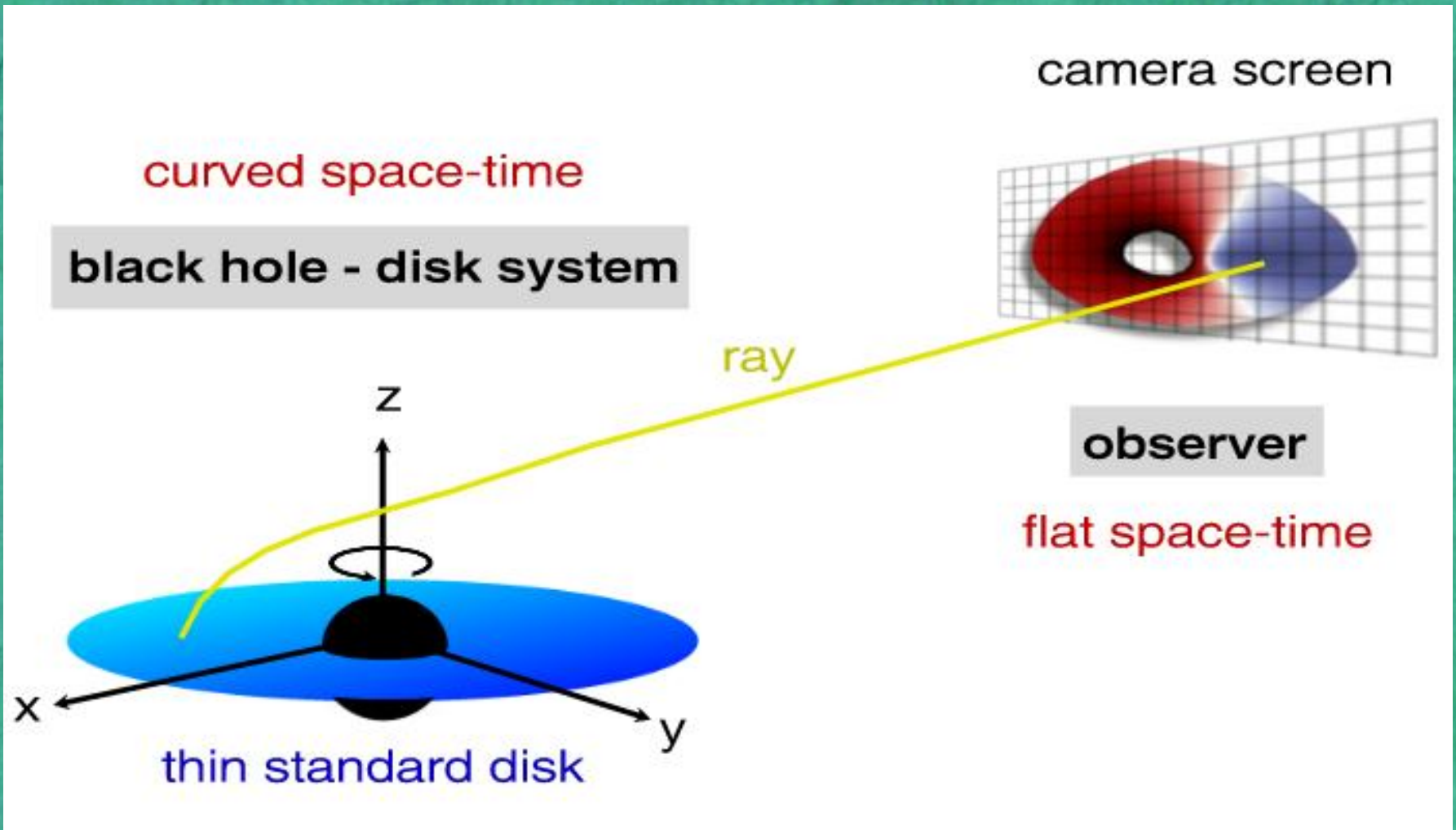
Kerr metric in Boyer-Lindquist co-ordinates $\{t, r, \Theta, \Phi\}$

$$g_{\mu\nu} = \begin{pmatrix} g_{tt} & 0 & 0 & g_{t\Phi} \\ 0 & g_{rr} & 0 & 0 \\ 0 & 0 & g_{\Theta\Theta} & 0 \\ g_{\Phi t} & 0 & 0 & g_{\Phi\Phi} \end{pmatrix} = \begin{pmatrix} -\alpha^2 + \omega^2 \tilde{\omega}^2 & 0 & 0 & -\omega \tilde{\omega}^2 \\ 0 & \rho^2 / \Delta & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ -\omega \tilde{\omega}^2 & 0 & 0 & \tilde{\omega}^2 \end{pmatrix}$$

$$g^{\mu\nu} = \begin{pmatrix} g^{tt} & 0 & 0 & g^{t\Phi} \\ 0 & g^{rr} & 0 & 0 \\ 0 & 0 & g^{\Theta\Theta} & 0 \\ g^{\Phi t} & 0 & 0 & g^{\Phi\Phi} \end{pmatrix} = \begin{pmatrix} -1/\alpha^2 & 0 & 0 & -\omega/\alpha^2 \\ 0 & \Delta/\rho^2 & 0 & 0 \\ 0 & 0 & 1/\rho^2 & 0 \\ -\omega/\alpha^2 & 0 & 0 & \frac{\alpha^2 - \omega^2 \tilde{\omega}^2}{\alpha^2 \tilde{\omega}^2} \end{pmatrix}$$

Numerical technique

Kerr ray tracing



Numerical technique

Geodesics equations in Kerr

- GR Lagrangian in Boyer-Lindquist co-ordinates
- Legendre transformation to Hamiltonian
- separability ansatz for Hamilton-Jacobi differential equation
- photon momenta follow from derivatives of action
- 4 conservatives:
 - energy E ,
 - mass μ ,
 - angular momentum J ,
 - Carter constant \mathbf{C} (Kerr-specific!)
- reduction to set of 4 1st order differential equations
- integraton of geodesics equations by
 - Runge-Kutta scheme (direct method)
 - elliptical integrals (*Fanton et al. 1997, A. Müller 2000*)
 - transfer functions (*Cunningham 1975, Bromley et al. 1997*)

Generalized Doppler factor g-factor

$$g \equiv \frac{\nu_{obs}}{\nu_{em}} = \frac{\hat{p}_{obs}^t}{\hat{p}_{em}^t}$$

definition in rest-frame

Carter momenta in ZAMO (1968)

$$\hat{p}^t = \gamma \left[p^{(t)} - v^{(j)} p_{(j)} \right]$$

Lorentz boost from ZAMO to
rest frame

$$g = \frac{\alpha}{\gamma \left[(1 - \omega\lambda) - \alpha v^{(r)} \frac{\sqrt{\mathcal{R}_0}}{\rho\sqrt{\Delta}} - \alpha v^{(\theta)} \frac{\sqrt{\Theta}}{\rho} - \alpha v^{(\Phi)} \frac{\lambda}{\tilde{\omega}} \right]}$$

$$= \frac{\alpha}{\gamma \left[1 - \alpha v^{(r)} \frac{\sqrt{\mathcal{R}_0}}{\rho\sqrt{\Delta}} - \alpha v^{(\theta)} \frac{\sqrt{\Theta}}{\rho} - \lambda\Omega \right]}$$

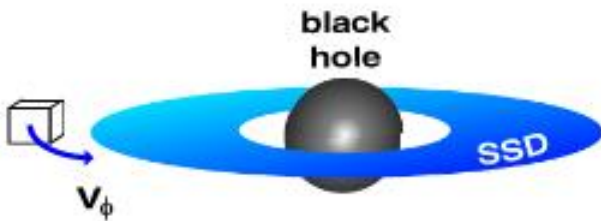
Plasma kinematics

Keplerian

$$v_\phi = v_{\text{Kepler}}$$

$$v_r = 0$$

$$v_\theta = 0$$



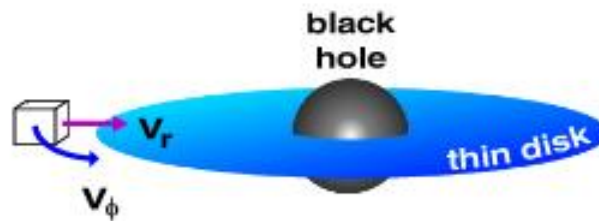
classical approach

non-Keplerian

$$v_\phi = v_{\text{Kepler}}$$

$$v_r \neq 0$$

$$v_\theta = 0$$



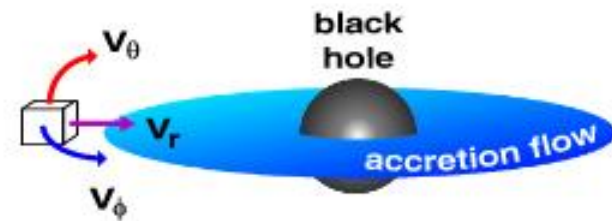
radial drift model

non-Keplerian

$$v_\phi \neq 0$$

$$v_r \neq 0$$

$$v_\theta \neq 0$$



radiative 3D-GRMHD

Accretion theory

Hydrodynamics and MHD

- co-existent and overlapping solutions available:
 - **ADAF** (*Advection-Dominated Accretion Flow*)
Narayan & Yi 1994
 - **ADIOS** (*Advection-Dominated Inflow-Outflow Solution*)
Blandford & Begelman 1999
 - **CDAF** (*Convection-Dominated Accretion Flow*)
Quataert & Gruzinov 2000
 - **ISAF** (*Ion-Supported Accretion Flow*)
Spruit & Deufel 2001
 - **TDAT** (*Truncated Disk – Advective Tori*)
Hujeirat & Camenzind 2001
 - **NRAF** (*Non-Radiative Accretion Flow*)
Balbus & Hawley 2002
- α - and β -disks
- complete parameter space investigation
- need for covariant radiative generalization!

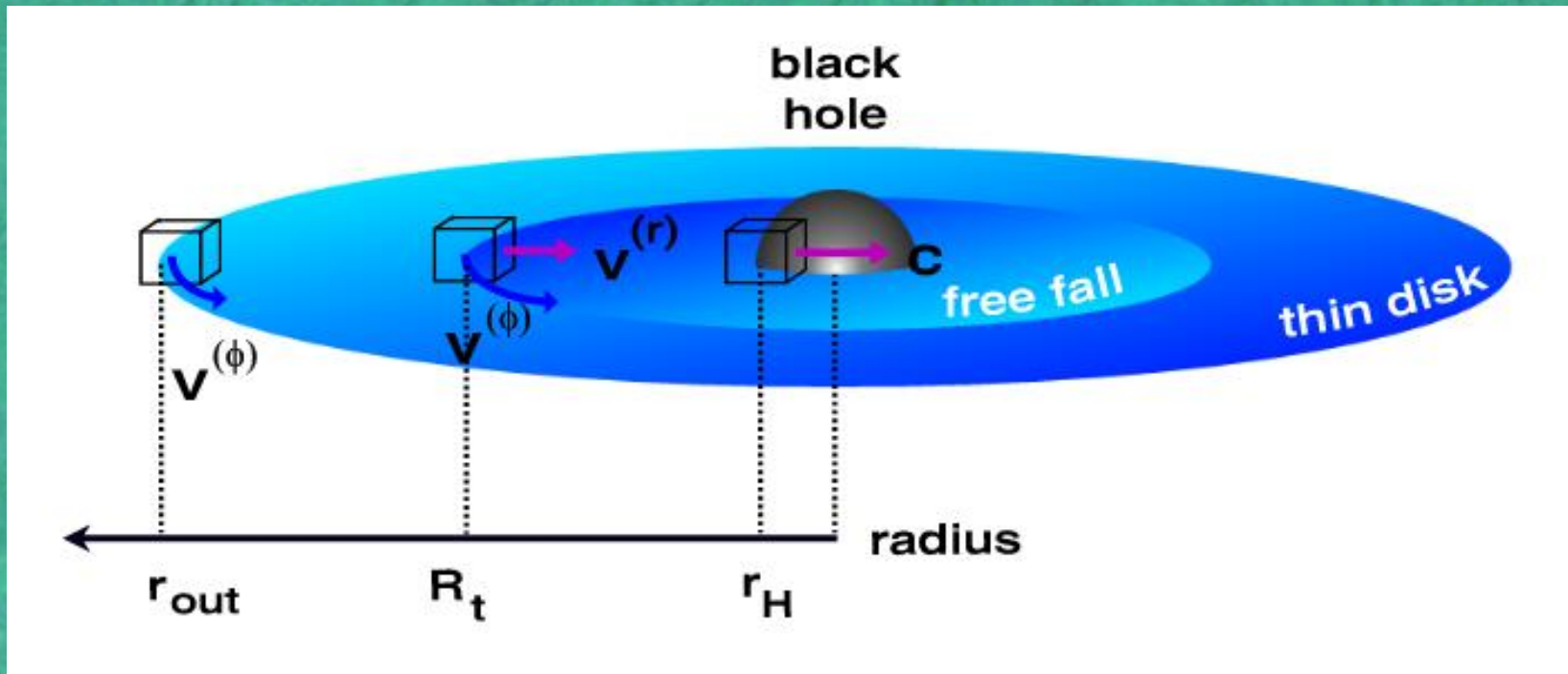
Radiation mechanisms

- **thermal emission**
 - ❖ single black body
 - ❖ multi-color black body (SSD)
- **Comptonization** (Kompaneets equation)
 - ❖ dominant global X-ray component
 - ❖ reprocessed soft photons from environment
 - ❖ corona: seed photon production for fluorescence
- **Synchrotron radiation**
 - ❖ radio emission
 - ❖ fast cooling of hot accretion flow on ms-scale
 - ❖ SSC (sub-mm bump)
 - ❖ SSA (dip feature)
- **bremsstrahlung**
 - ❖ launch of outflow (disk wind, Poynting flux)

→ Covariant generalization: GR radiation transfer!

Radial drift model

Truncation and free-fall



Truncated Standard accretion Disks (TSD) due to efficient radiative cooling. Disk cuts off at R_t , not at r_{ms} (cp. SSD) depending on radiative accretion theory (accretion rate, cooling, conduction).

(Hujeirat & Camenzind 2000)

Radial drift model

Velocity field in ZAMO frame

ZAMO velocities

$$v^{(\Phi)} = \tilde{\omega} \left(\frac{\Omega - \omega}{\alpha} \right)$$

$$v^{(r)} = \frac{\sqrt{\mathcal{R}}}{\Sigma(1 - \omega\lambda)}$$

angular frequencies

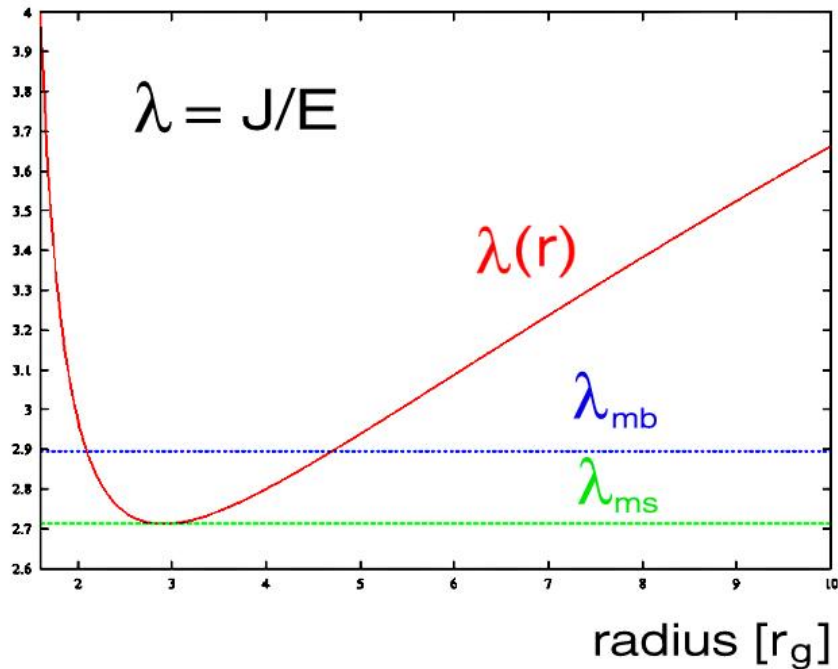
$$\Omega_K = \pm \frac{\sqrt{M}}{\sqrt{r^3} \pm a\sqrt{M}}$$

$$\Omega = \Omega_{in} = \omega + \frac{\alpha^2}{\tilde{\omega}^2} \frac{\lambda_{ms}}{1 - \omega\lambda_{ms}}$$

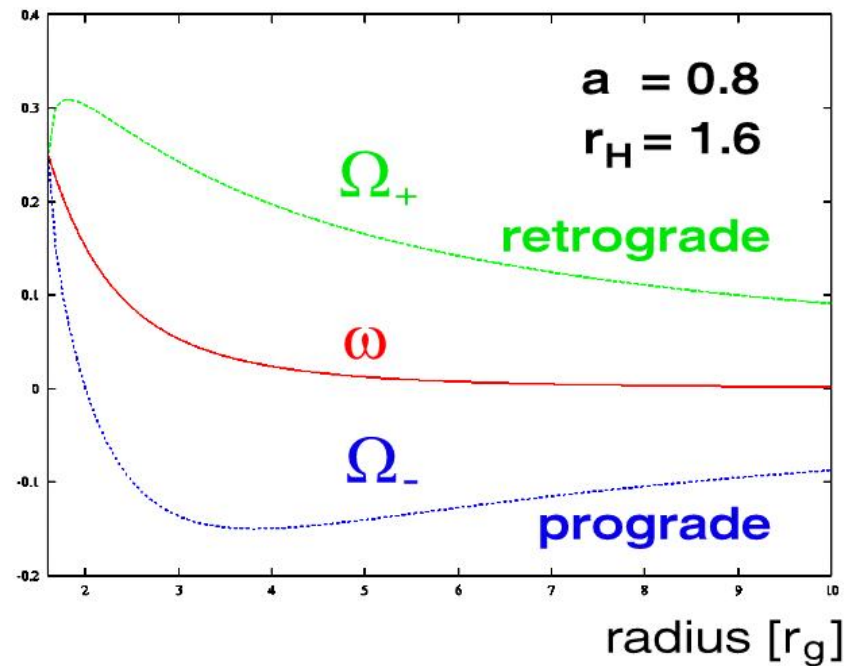
$$\lambda_{ms} = \frac{\tilde{\omega}_{ms}^2 (\Omega_{K,ms} - \omega_{ms})}{\alpha_{ms}^2 + \omega_{ms} \tilde{\omega}_{ms}^2 (\Omega_{K,ms} - \omega_{ms})}$$

Radial drift model

Parameter restrictions

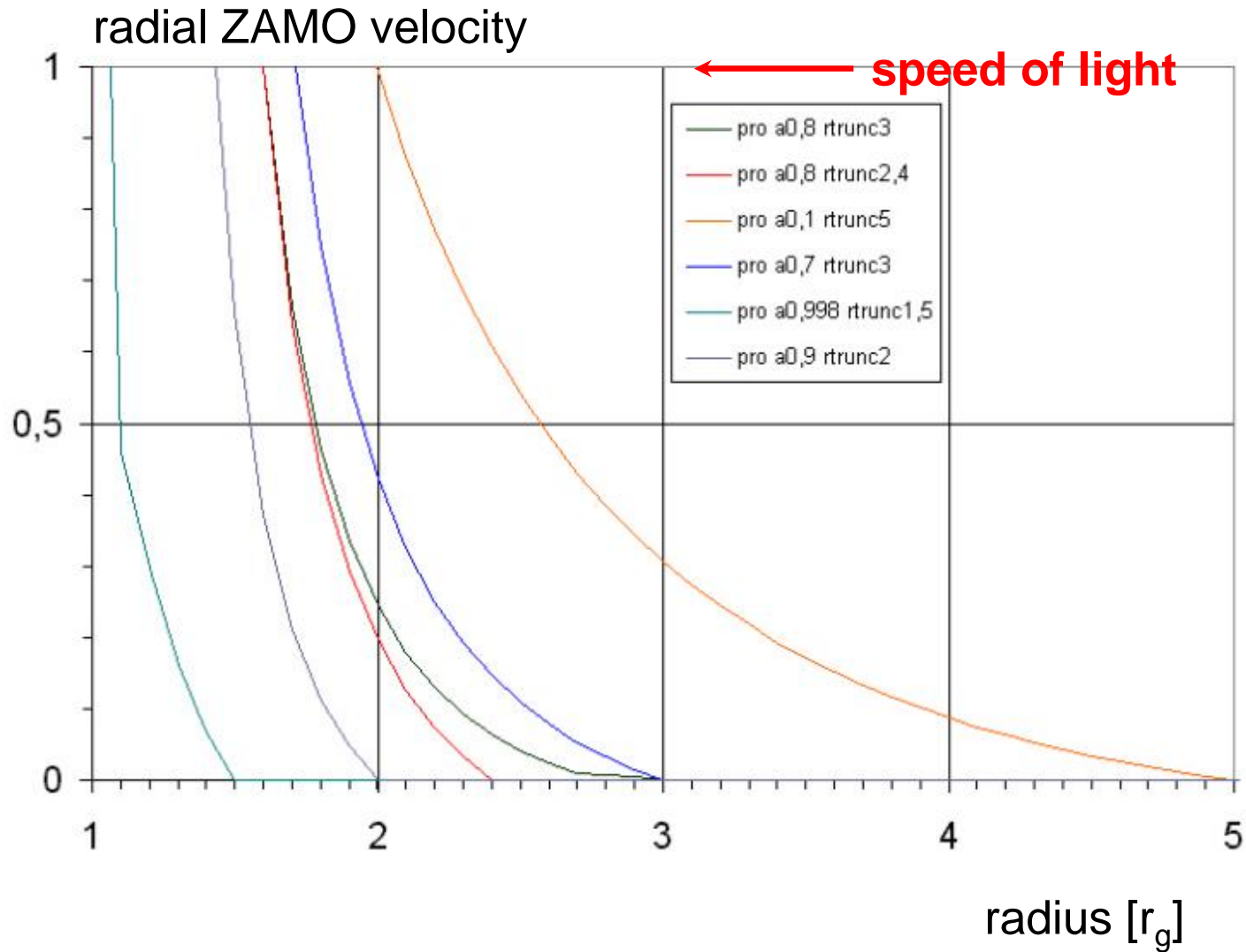


Specific angular momentum λ has chosen between λ_{ms} and λ_{mb} .

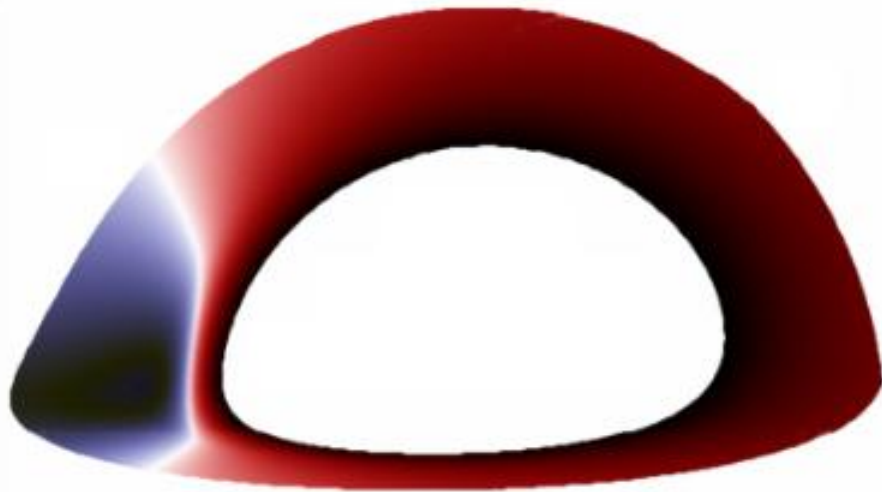


Only region between Ω_+ and Ω_- is allowed (time-like trajectories).

Radial drift models

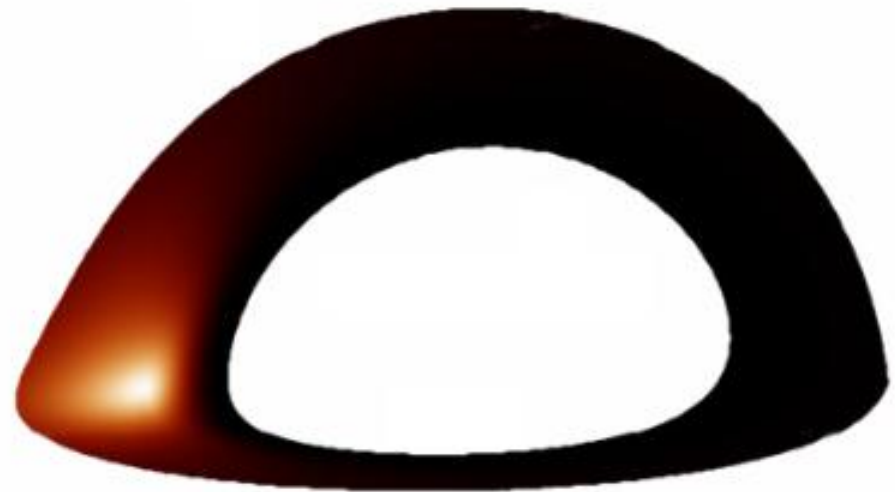


Rendered disk images g-factor and emission



g-factor

$$\begin{aligned} a &= 0.8 \\ i &= 80^\circ \\ r_{\text{in}} &= r_{\text{H}} = 1.6 \\ r_{\text{out}} &= 4.6 \\ R_t &= 3.0 \\ \sigma_r &= 3.0 \end{aligned}$$



emission

Disk emission

Relativistic effects

**Equatorial emitting ring with
orbiting free-falling matter
at high inclination**

rotating black hole
(Kerr solution)

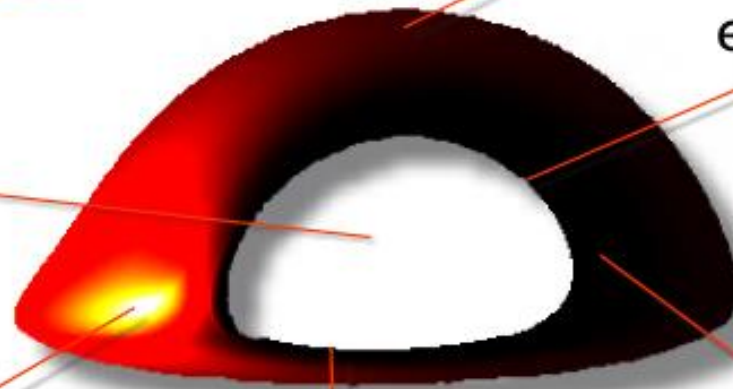
front beaming
(enhanced emission)

gravitational redshift

lensed disk
segment

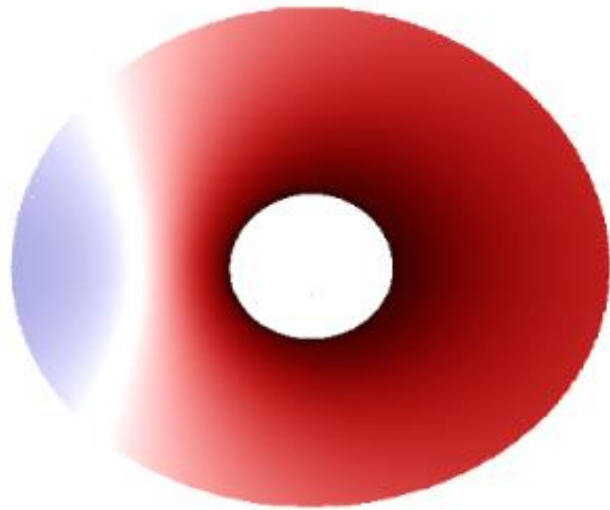
event horizon

back beaming
(reduced emission)

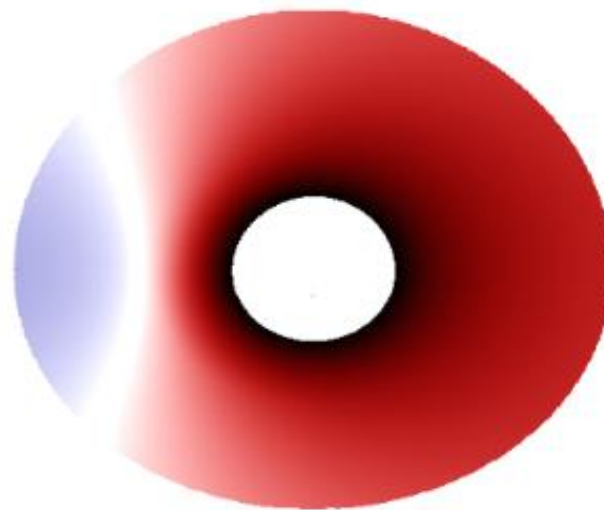


Radial drift model

g-factor: Keplerian vs. Drift



pure Keplerian



*Keplerian plus
radial drift*

$$\begin{aligned} a &= 0.1 \\ i &= 40^\circ \\ r_{\text{in}} = r_{\text{H}} &= 1.996 \\ r_{\text{out}} &= 10.0 \\ R_{\text{t}} &= 5.0 \end{aligned}$$

Radial drift model

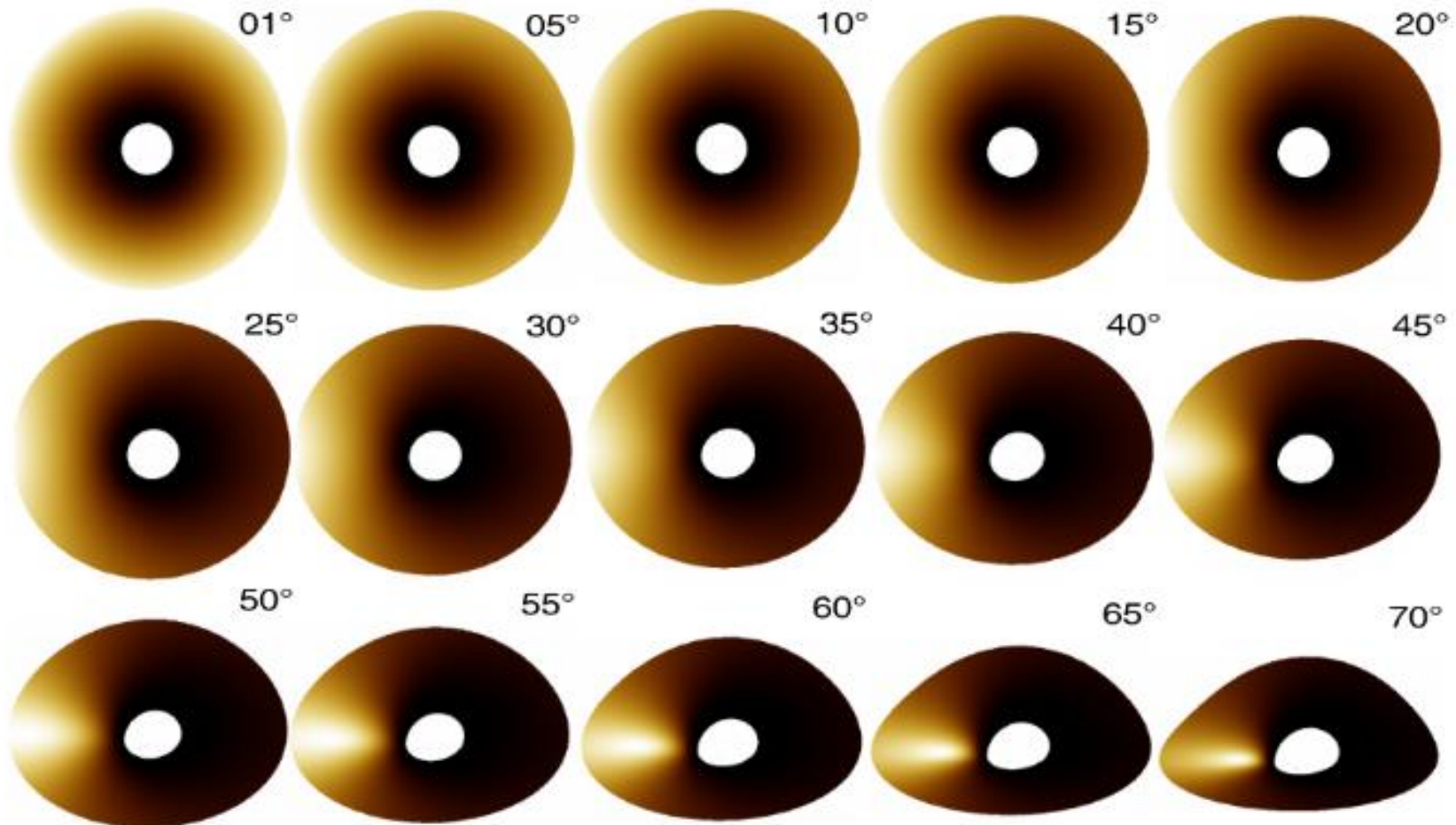
Implications

- adequate consideration of **accreted inflow**
- **truncation** softens the „*evidence for Kerr*“- argument, because R_t replaces r_{ms} . Coupling between r_{in} and r_{ms} is lost!
- **gravitational redshift** is enhanced!
- emission line shape does not change dramatically compared with pure Keplerian: only ***red wing effects***
- **poloidal motion** still neglected!
- awaiting new accretion theory: **covariance**
- follow *Armitage & Reynolds (2003)* approach: **couple** line emission to accretion model

Disk emission

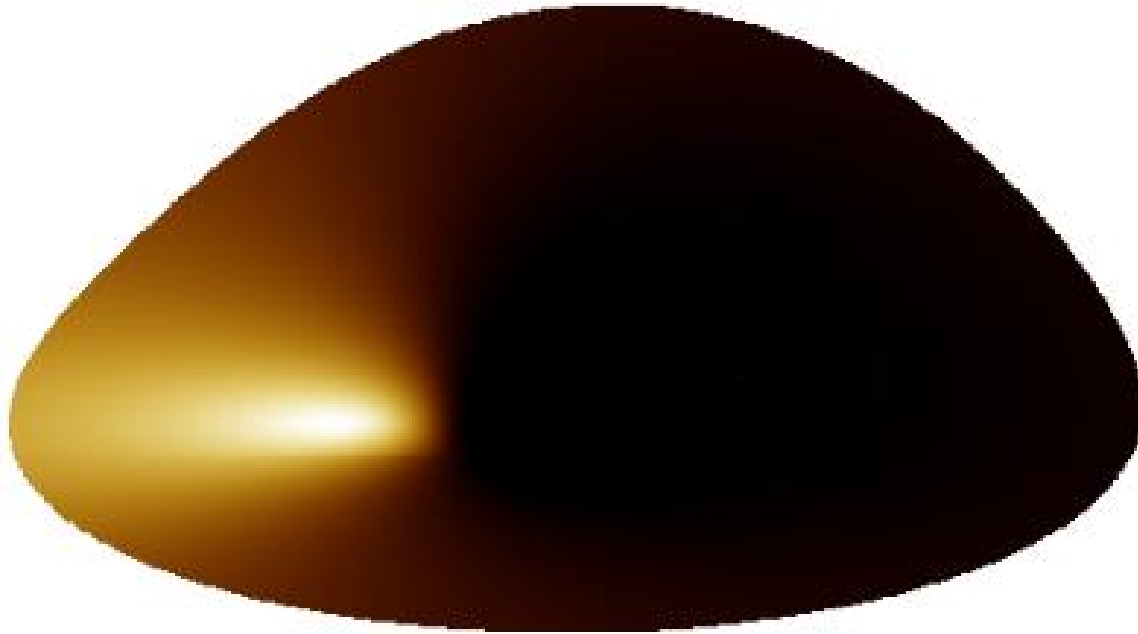
Inclination study with g^4

The **Shadow** of Kerr Black Holes



Black hole shadow

The **proximity**
of light and darkness



$$r_{\text{in}} = 1.0015 \quad a = 0.9999999$$
$$r_{\text{out}} = 10.0 \quad i = 70^\circ$$

Strong gravitational
redshift

horizon:
 $g = 0$

Flux integral folds g
in high power with
emissivity.

g^4 – distribution
suppresses *any emission*
near black holes!

„Shadow“
by Falcke et al. 2000

Relativistic emission line Calculation

$$F^{obs} = \int_{image} d\Xi I_{\nu}^{obs}$$

general spectral flux integral

$$I_{\nu}^{obs} = g^3 \hat{I}_{\nu}^{em}$$

using Lorentz invariant
(Misner 1973)

$$\hat{F}_{\nu}^{em} = \pi \hat{I}_{\nu}^{em} = \epsilon(r) \delta(\nu_{em} - \nu_0)$$

assume line shape in rest frame:
 δ -distribution

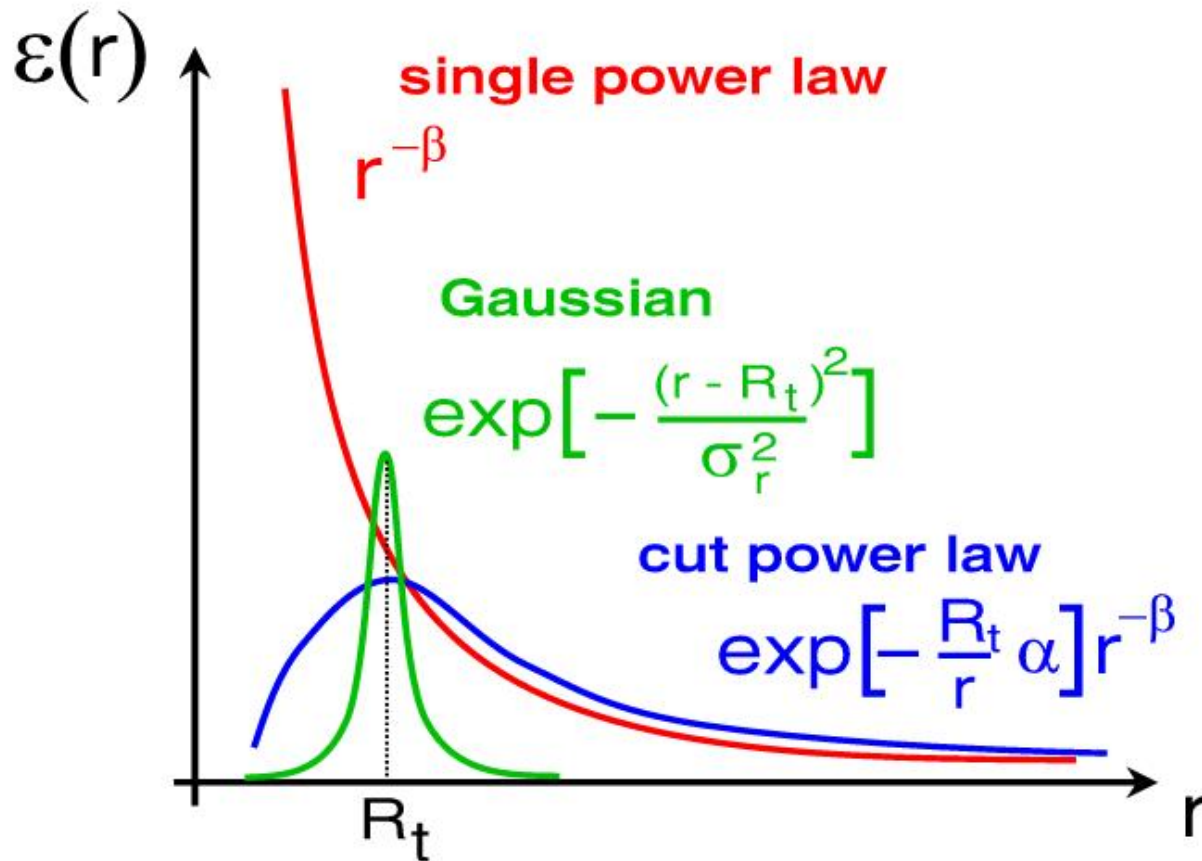
fold radial emissivity profile

- single power law
- double or broken power law
- Gaussian
- cut-power law

$$F_{obs}(E_{obs}) = \int_{image} \epsilon(r) g^4 \delta(E_{obs} - gE_0) d\Xi$$

evaluate tuple $\{g, \Delta\Omega, r\}$ on
each pixel and sum over pixels!

Radial emissivity profiles



emissivities

single power law
(Page & Thorne 1974)

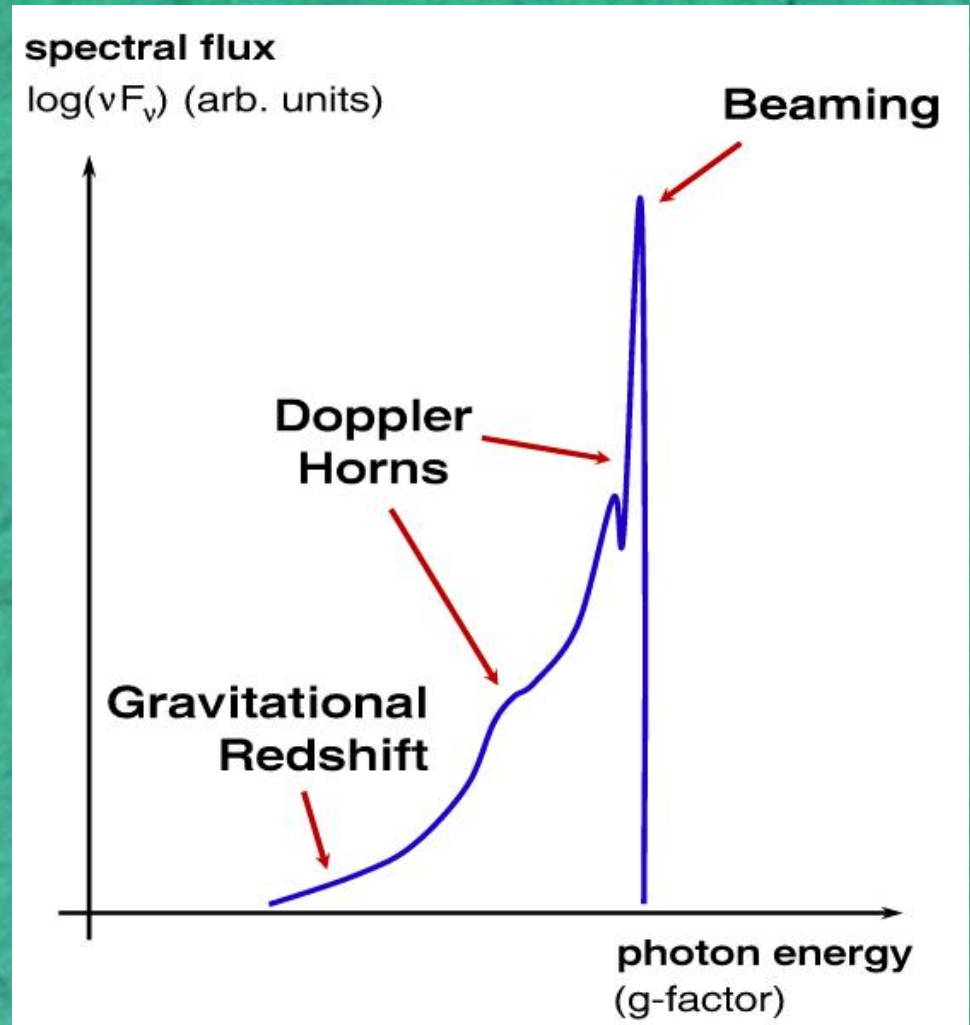
double or broken
power law

Gaussian,
cut-power law
(Müller & Camenzind 2003)

Line features

Imprints of relativistic effects

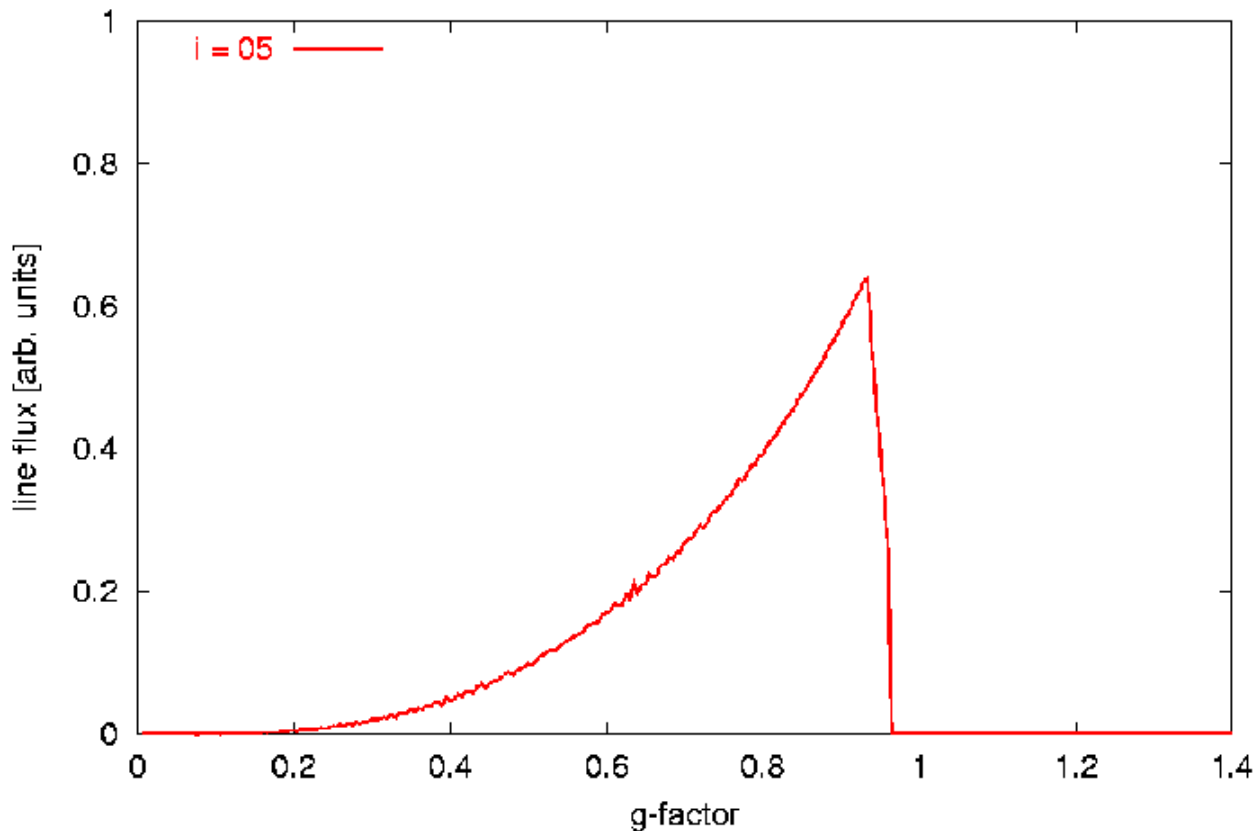
- Doppler (Newtonian)
- Beaming (SR)
- Gravitational redshift (GR)



Line studies

Inclination

Relativistic Emission Line at $a = 0.999999$ with variable inclination i



Parameters:

$a = 0.999999$

$i = 5^\circ \dots 70^\circ$

$r_{\text{in}} = r_{\text{ms}} = 1.0015$

$r_{\text{out}} = 30.0$

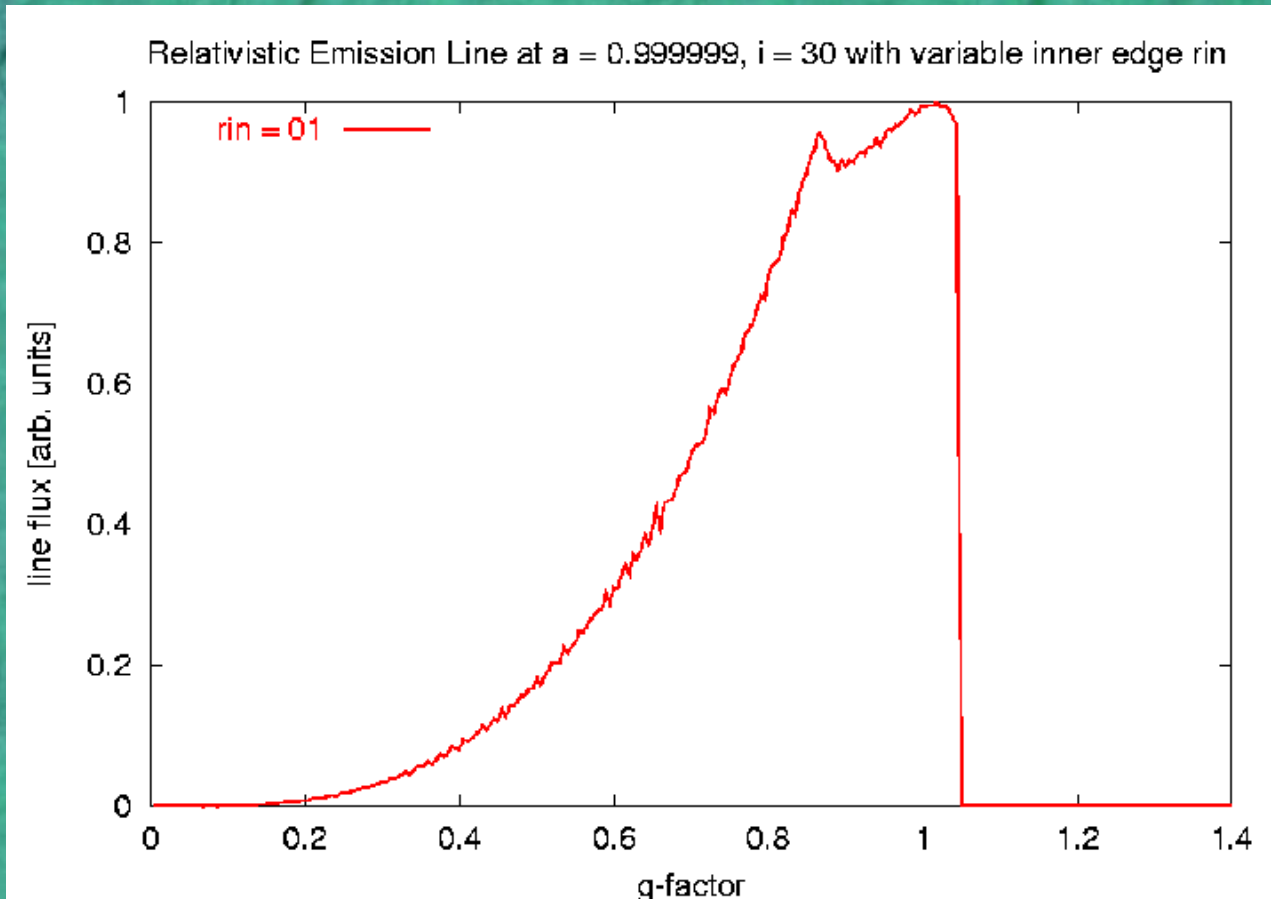
single power law
emissivity

pure rotation,
no drift

Blue edge shifts!
Enhanced Beaming!
Doppler effect

Line studies

Inner disk edge



Parameters:

$a = 0.9999999$

$i = 30^\circ$

$r_{in} = 1...28$

$r_{out} = 30.0$

single power law

emissivity

pure rotation,

no drift

Static blue edge!

Red wing vanishes!

Doppler effect

end: Newtonian

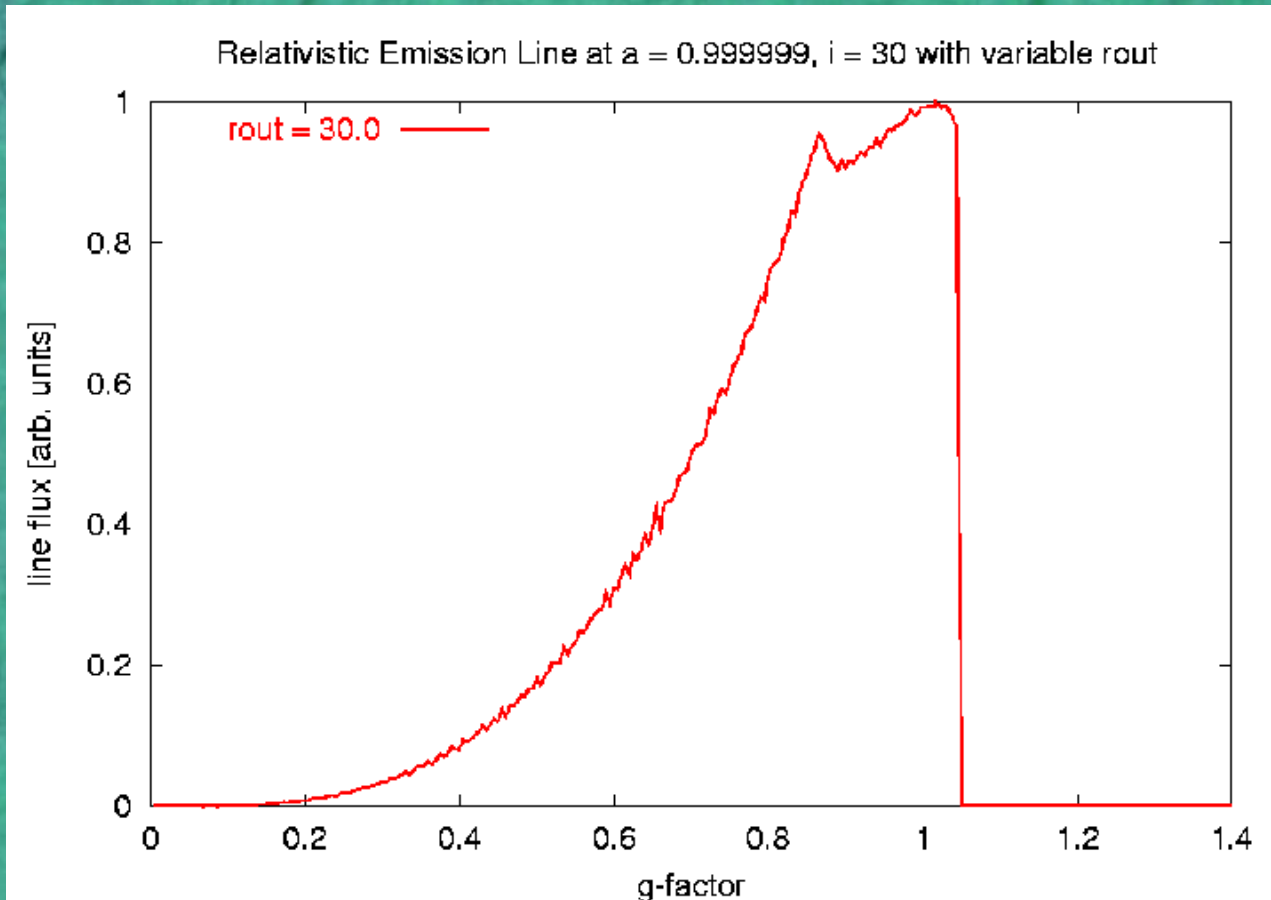
Space-time curvature

is negligible at

radii $\sim 20 r_g$!!!

Line studies

Outer disk edge



Parameters:

$a = 0.999999$

$i = 30^\circ$

$r_{in} = 1.0015$

$r_{out} = 30 \dots 1.5$

single power law
emissivity

pure rotation,
no drift

Static red edge!

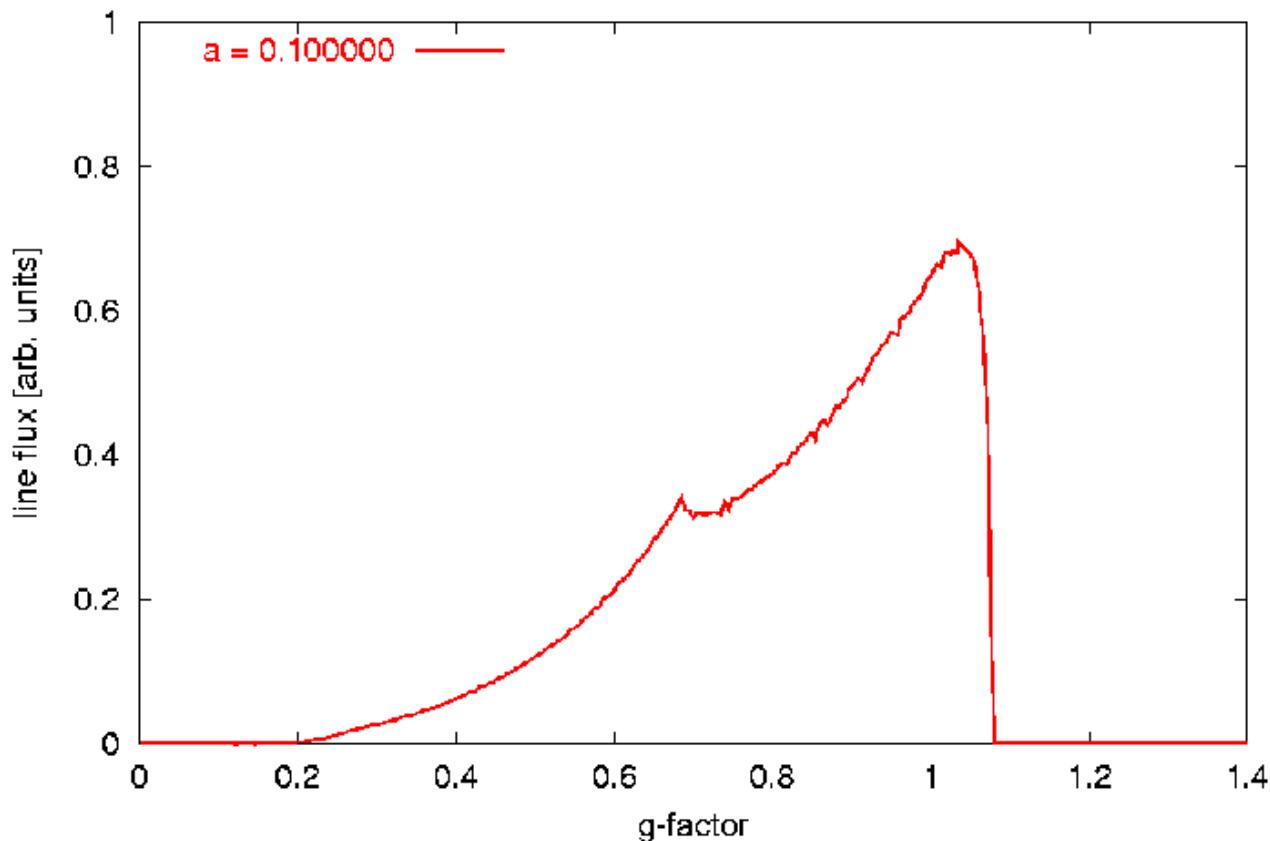
Beaming vanishes!

Doppler effect

Line studies

Kerr parameter

Relativistic Emission Line at $i = 40$ with variable Kerr parameter a



Parameters:

$a = 0.1 \dots 0.999999$

$i = 40^\circ$

$r_{in} = r_{ms}$

$r_{out} \sim 10.0$ decreasing

constant emitting area!

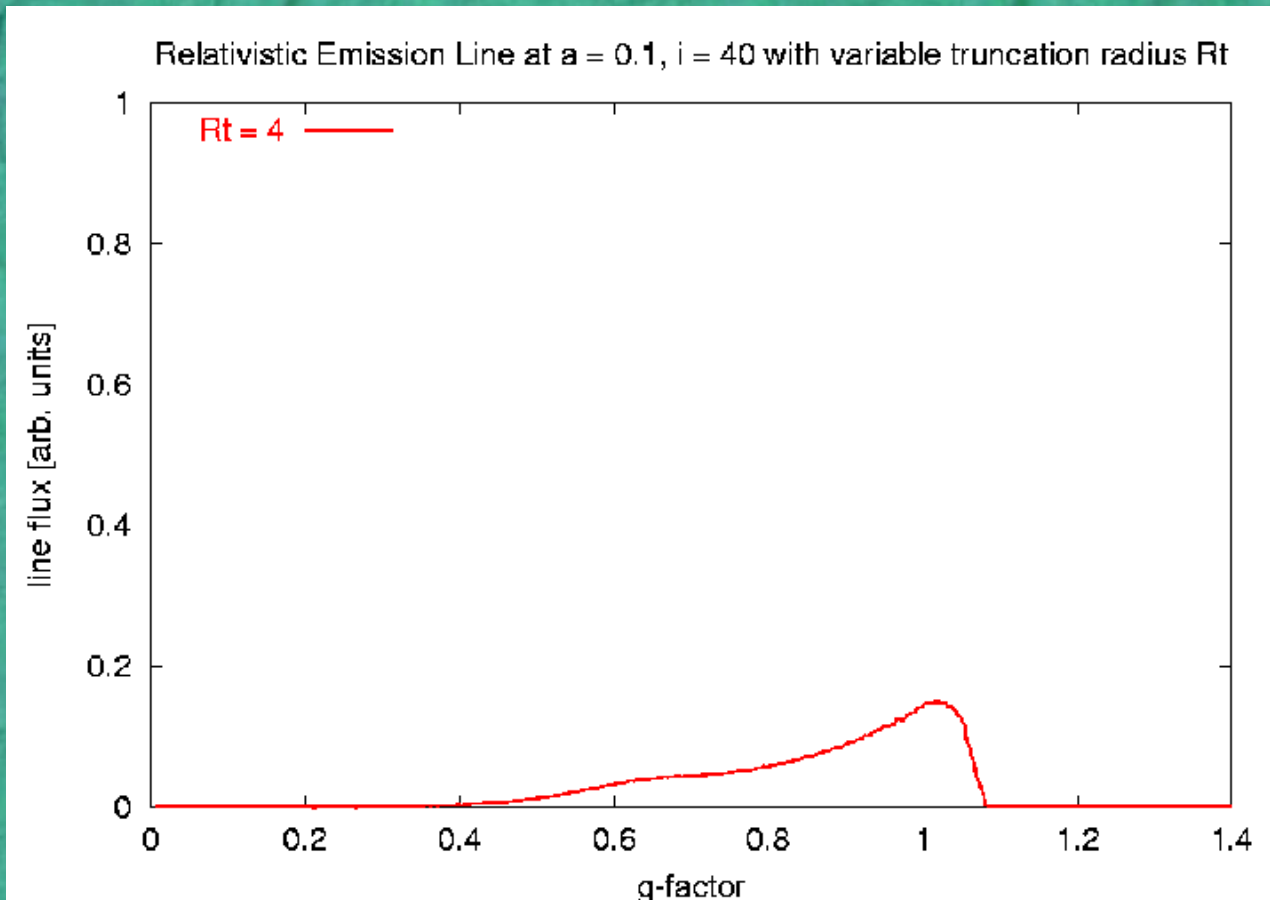
single power law emissivity

pure rotation,
no drift

Beaming increases due
to increasing *frame-
dragging effect!*

Line studies

Truncation radius



Parameters:

$$a = 0.1$$

$$i = 40^\circ$$

$$r_{\text{in}} = r_H = 1.995$$

$$r_{\text{out}} = 30.0$$

$$R_t = 4 \dots 8$$

$$\sigma_r = 0.4 R_t$$

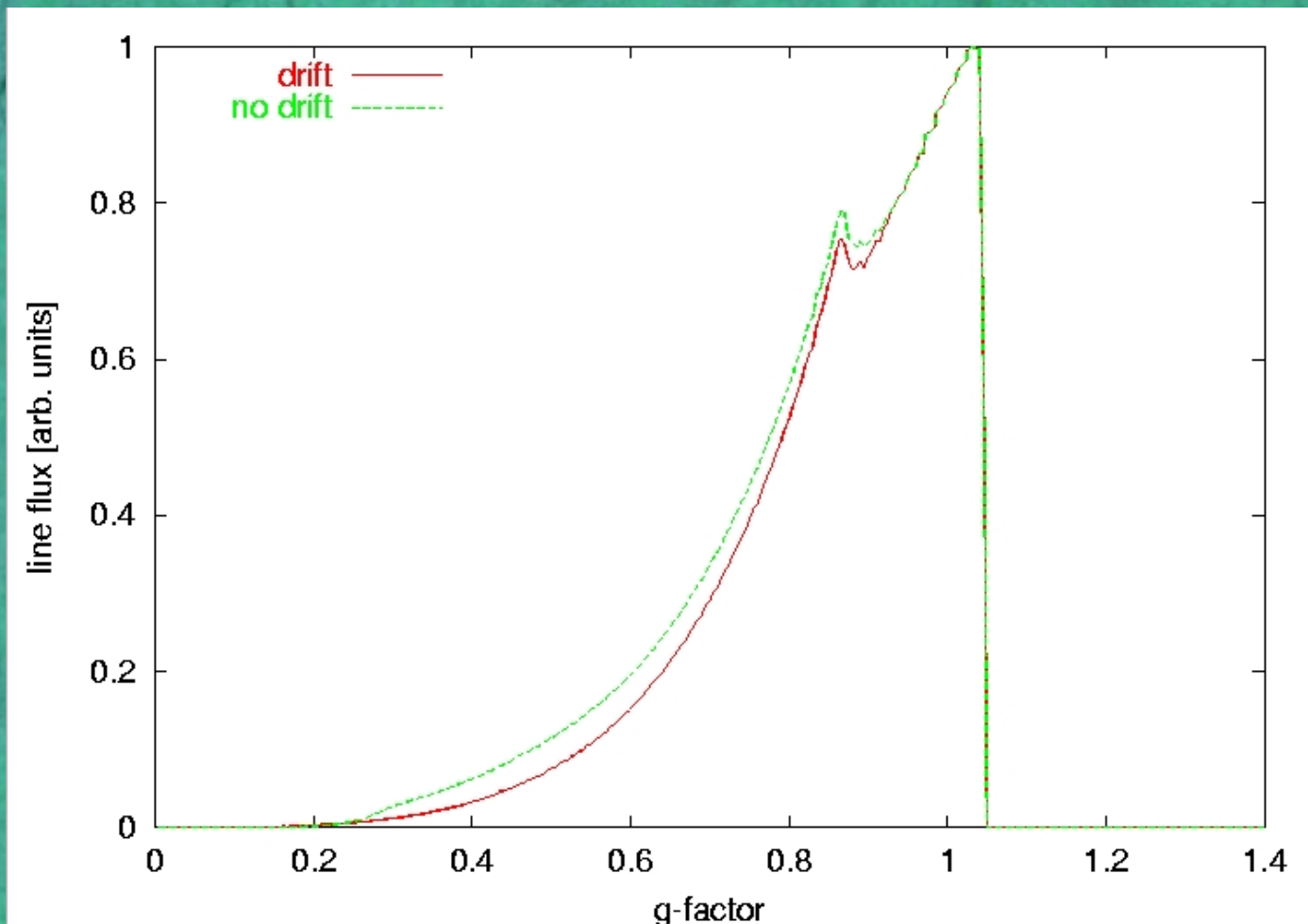
Gaussian emissivity
couples to R_t

non-Keplerian:
rotation plus drift!

Gravitational redshift
decreases with radius!
Enhanced Beaming!
Doppler effect

Line studies

Drift + rotation vs. pure rotation



Parameters:

$a = 0.001$

$i = 30^\circ$

$r_{in} = r_H = 2.0$

$r_{out} = 30.0$

$R_t = 6$

single power law
emissivity

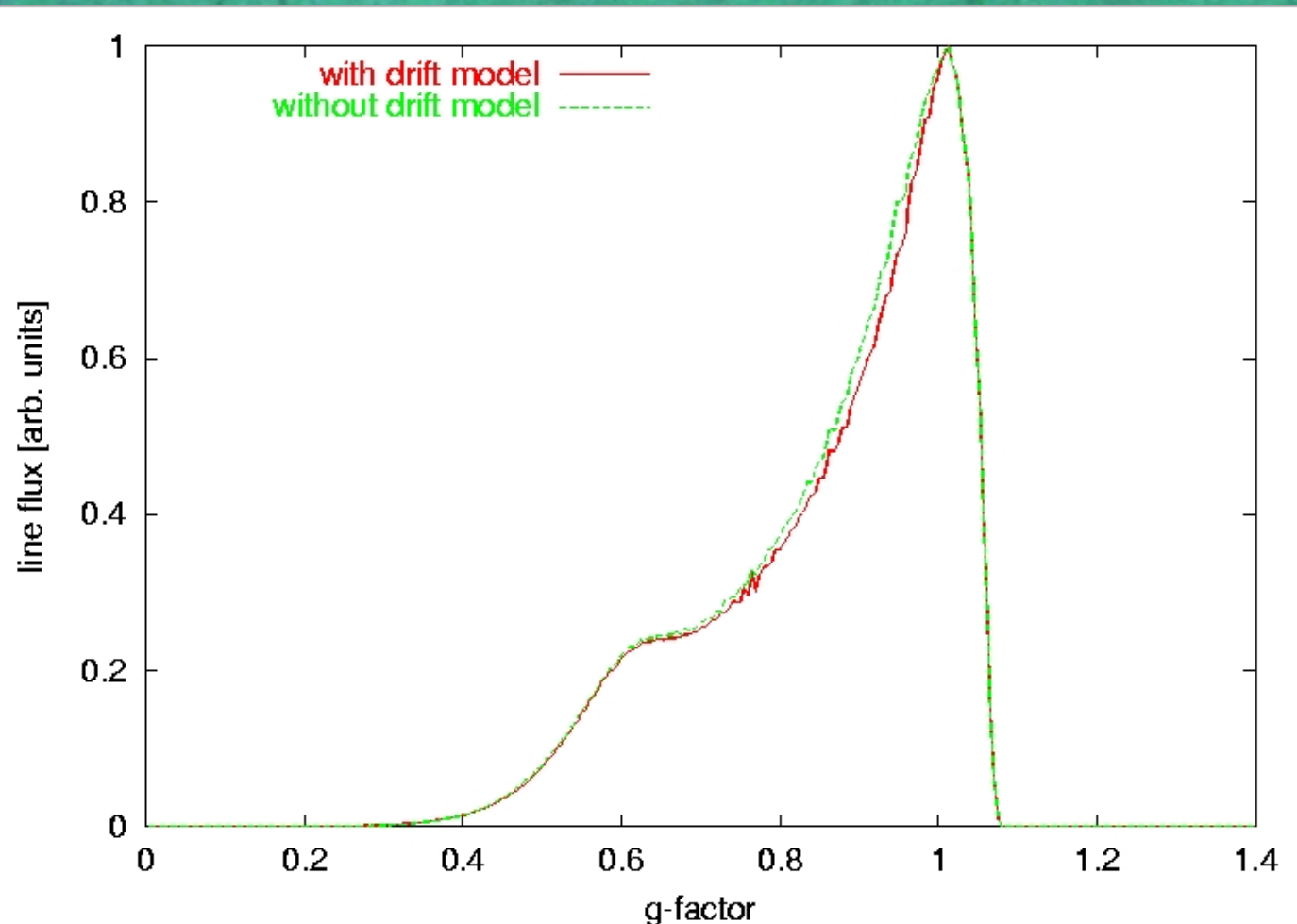
pure Keplerian

non-Keplerian:
rotation plus drift!

Drift causes
enhanced
gravitational redshift
and reduces red wing
flux!

Line studies

Drift + rotation vs. pure rotation



Parameters:

$$a = 0.1$$

$$i = 40^\circ$$

$$r_{\text{in}} = r_{\text{H}} = 1.995$$

$$r_{\text{out}} = 10.0$$

$$R_t = 5$$

$$\sigma_r = 0.4 R_t$$

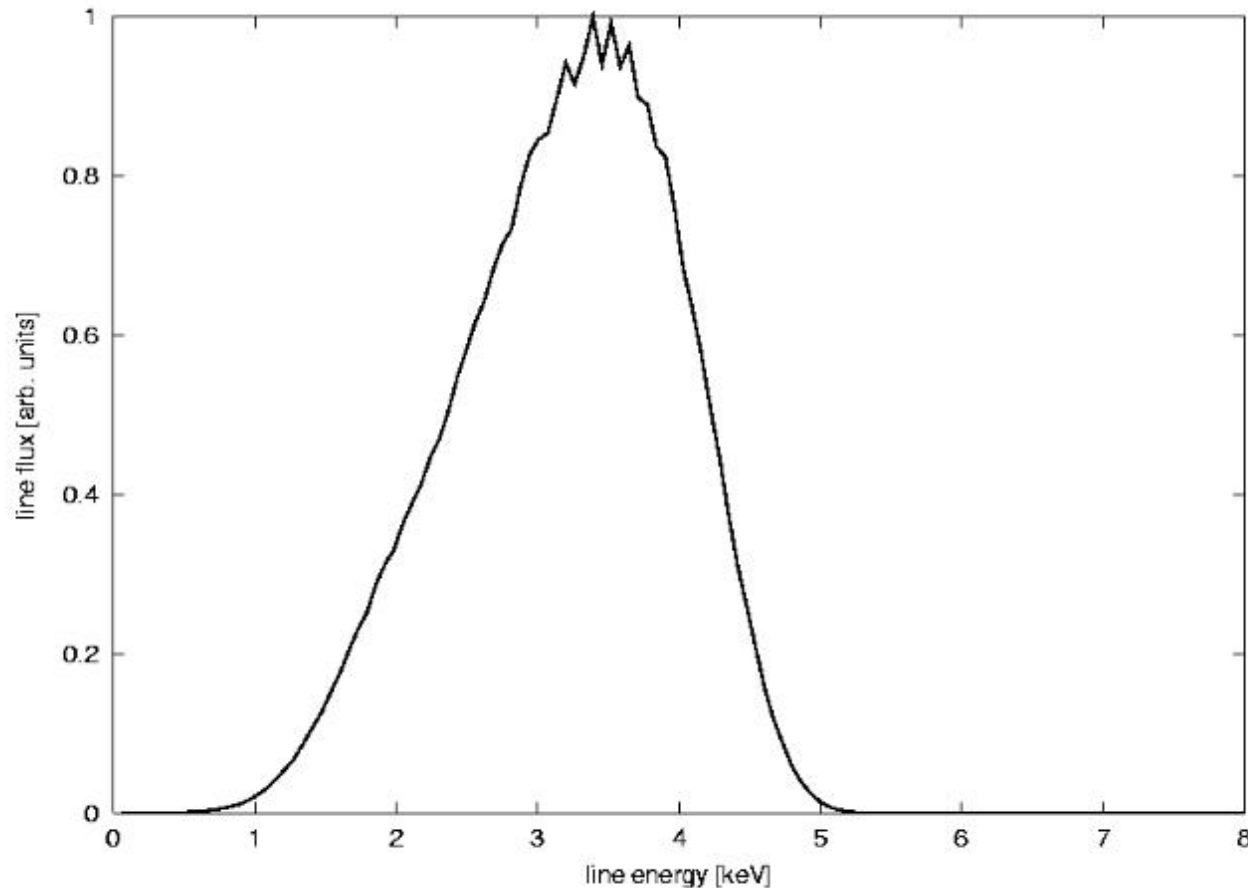
Gaussian emissivity
couples to R_t

pure Keplerian

non-Keplerian:
rotation plus drift!

Gravitational redshift
causes red wing
differences!

Line suppression „Shadowed lines“



Parameters:

$$a = 0.998$$

$$i = 30^\circ$$

$$r_{\text{in}} = r_{\text{H}} = 1.06$$

$$r_{\text{out}} = 30.0$$

$$R_t = 1.5$$

$$\sigma_r = 0.4$$

Gaussian emissivity

non-Keplerian:

rotation + drift

peak at ~ 3 keV

high redshift!

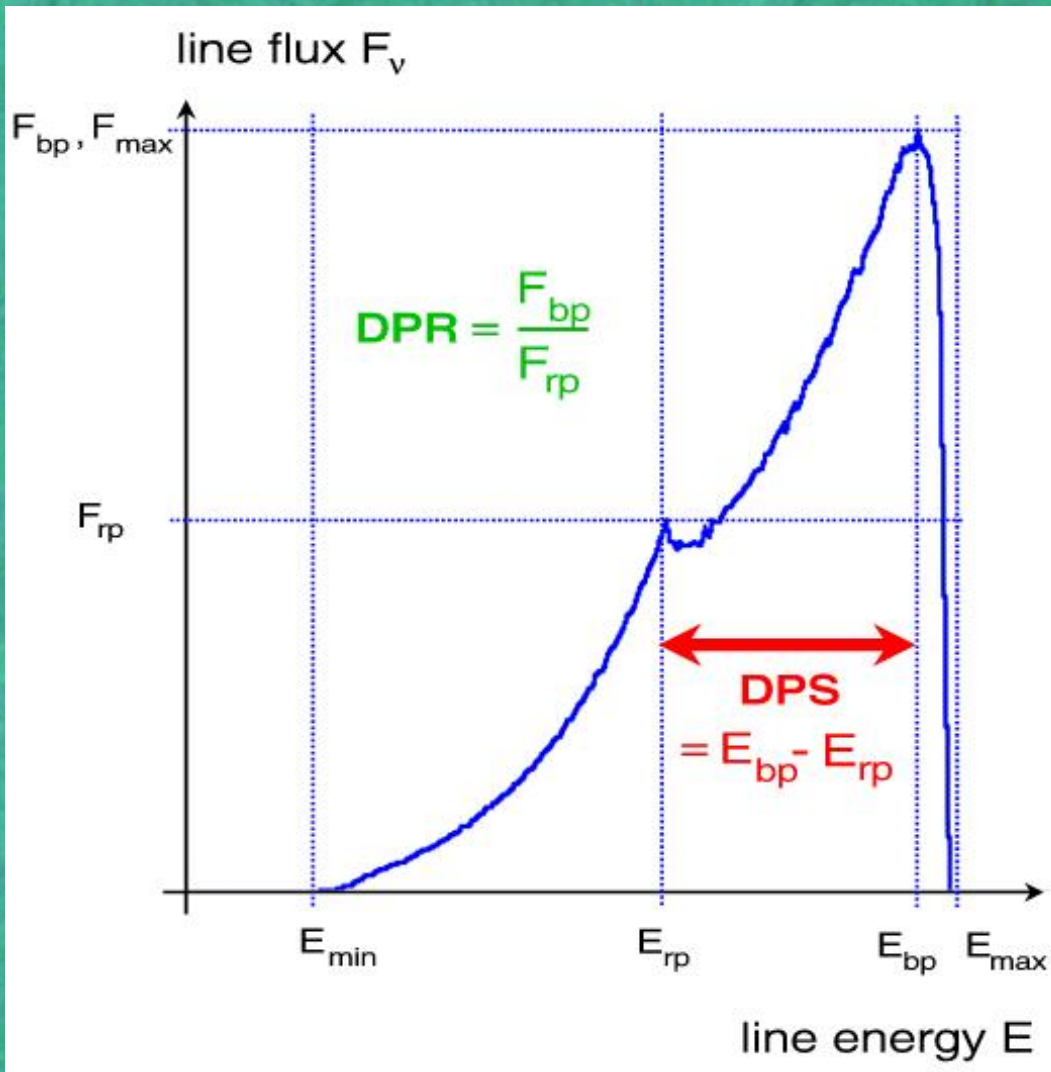
(„unphysical“

line: consider

fluorescence

restrictions)

Line criteria



DPR

Doppler Peak Ratio

DPS

Doppler Peak Spacing

(relative quantities!)

(Müller & Camenzind 2003)

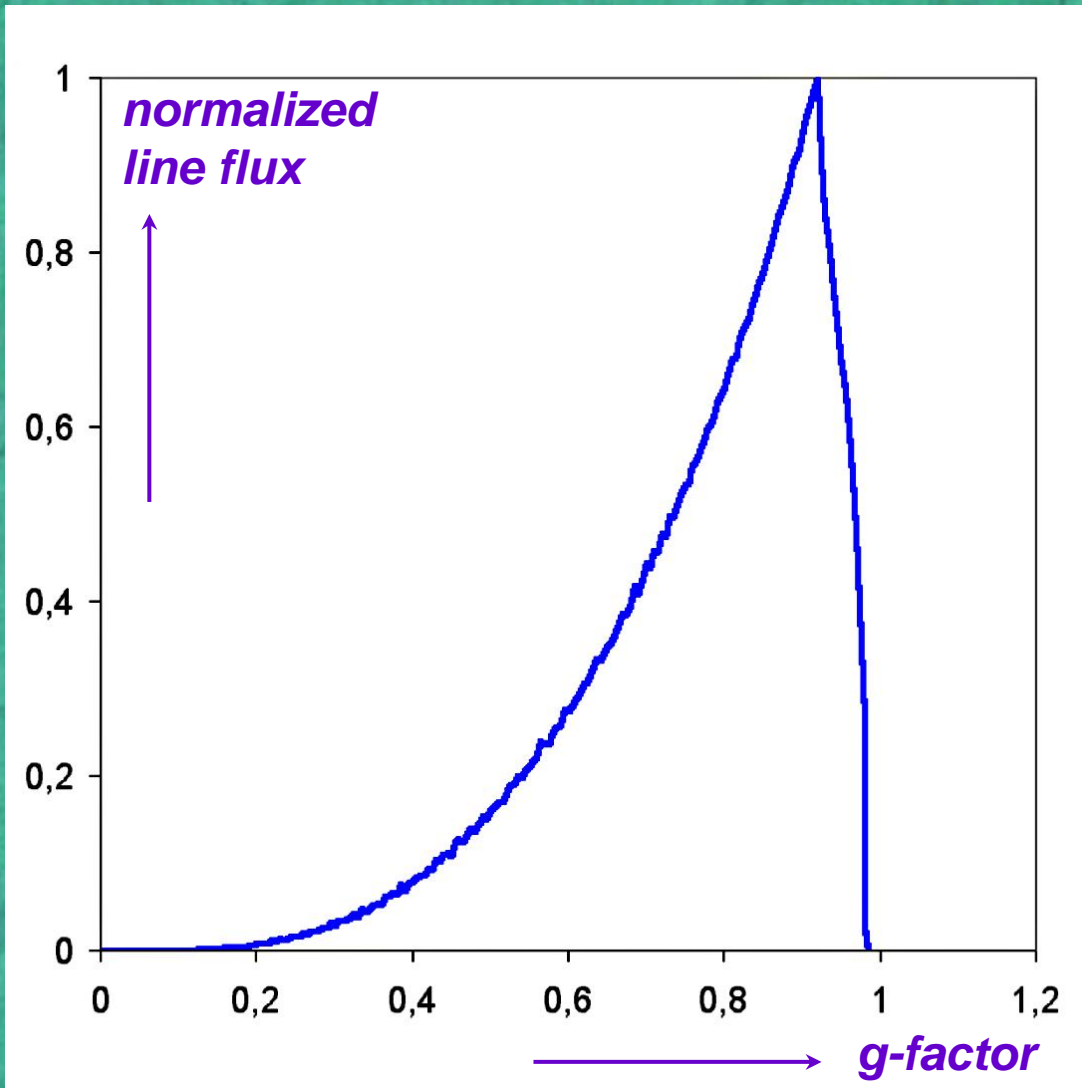
Line classification

Proposed nomenclature

- topological criterion:
 - triangular
 - bumpy
 - double-horned
 - double-peaked
 - shoulder-like
- pre-selection of parameters possible
- pre-classification of observed lines
- unification scheme of AGN

Line classification

Triangular



Parameters:

$$a = 0.999999$$

$$i = 10^\circ$$

$$r_{\text{in}} = 1.0015$$

$$r_{\text{out}} = 30.0$$

$$\beta = 3.0$$

single power law

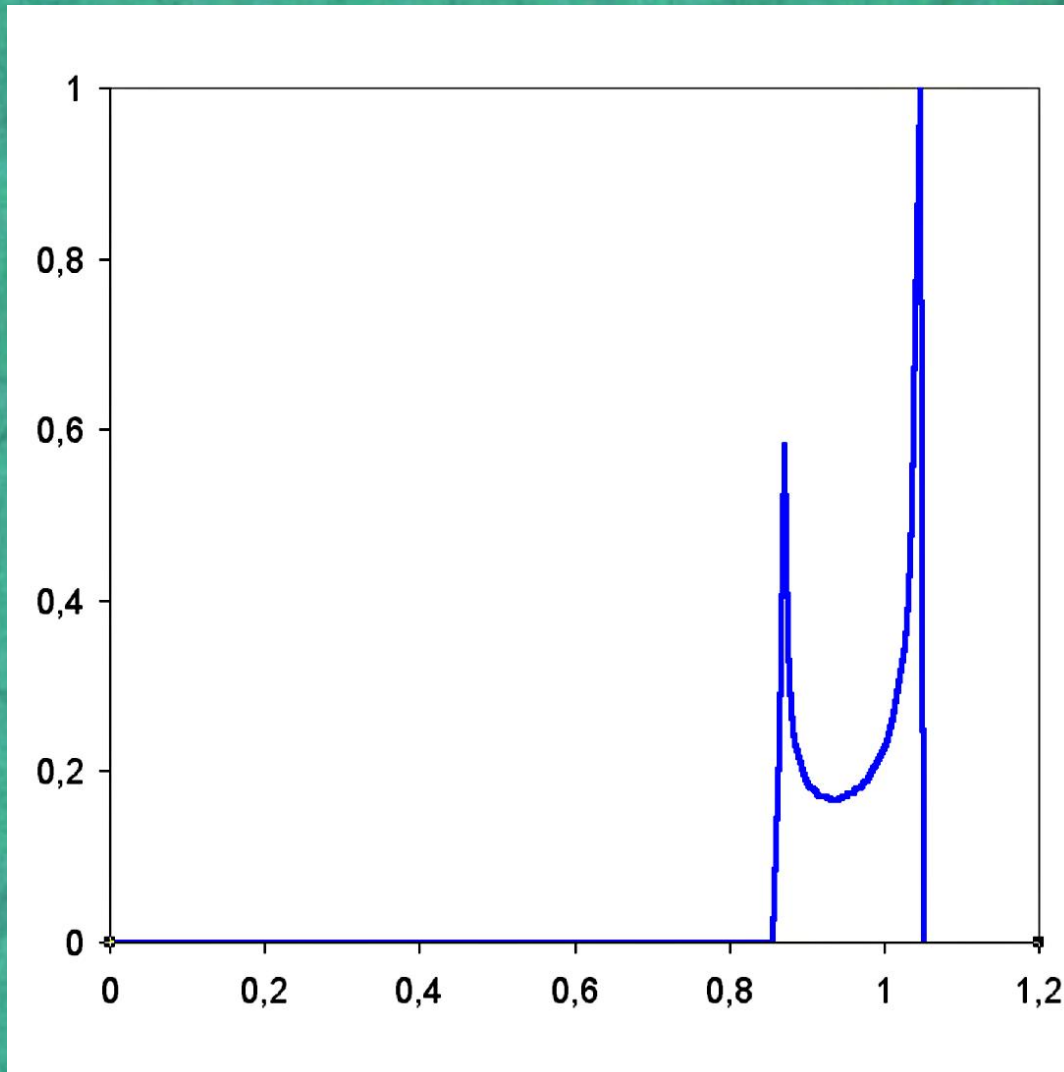
Keplerian

typical:

low inclination, Doppler reduced

Line classification

Double-peaked



Parameters:

$$a = 0.999999$$

$$i = 30^\circ$$

$$r_{\text{in}} = 28.0$$

$$r_{\text{out}} = 30.0$$

$$\beta = 3.0$$

single power law

Keplerian

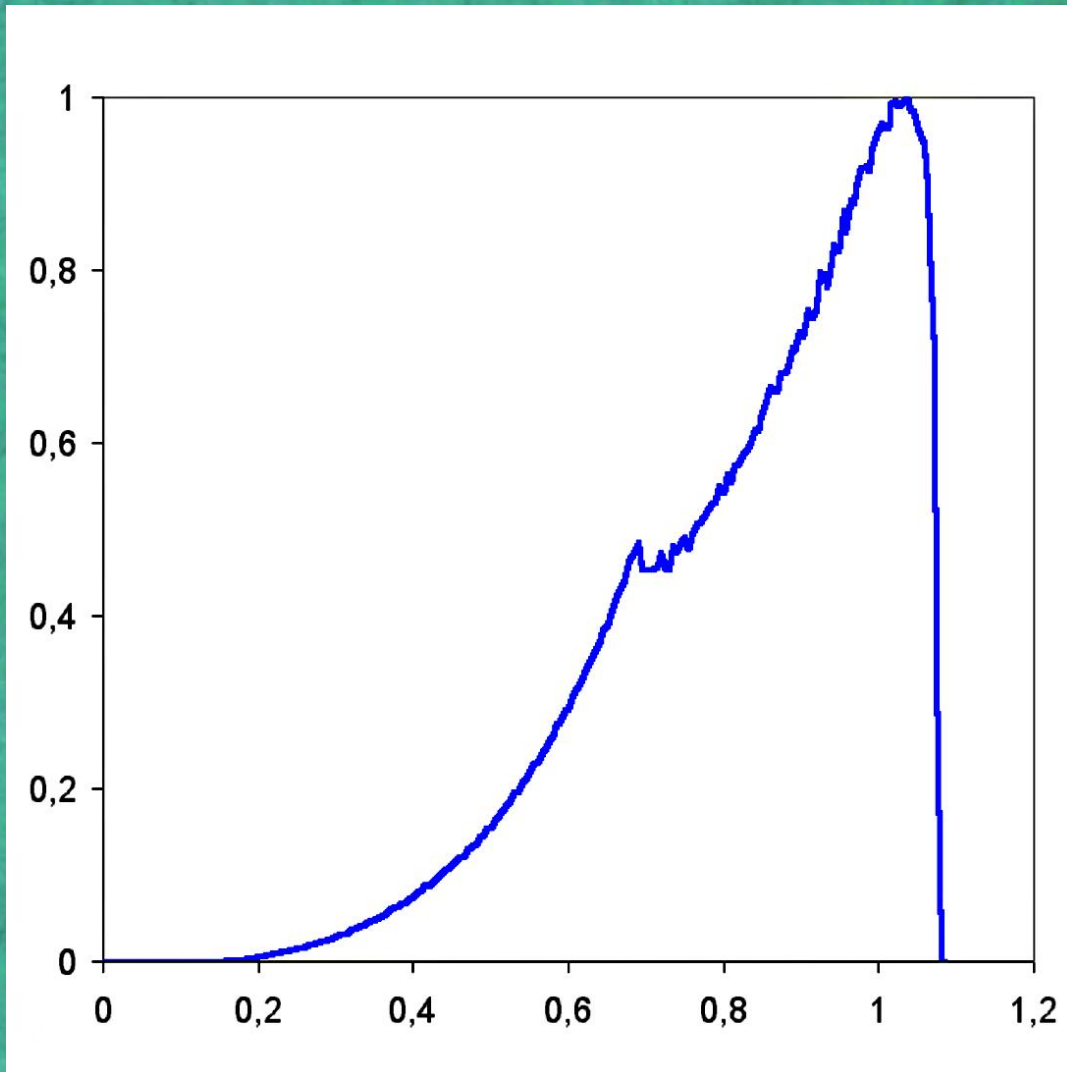
typical:

medium2high inclination,
asymptotically

flat metric, no GR effects

Line classification

Double-horned



Parameters:

$$a = 0.4$$

$$i = 40^\circ$$

$$r_{\text{in}} = 1.9165$$

$$r_{\text{out}} = 9.9846$$

$$\beta = 3.0$$

single power law

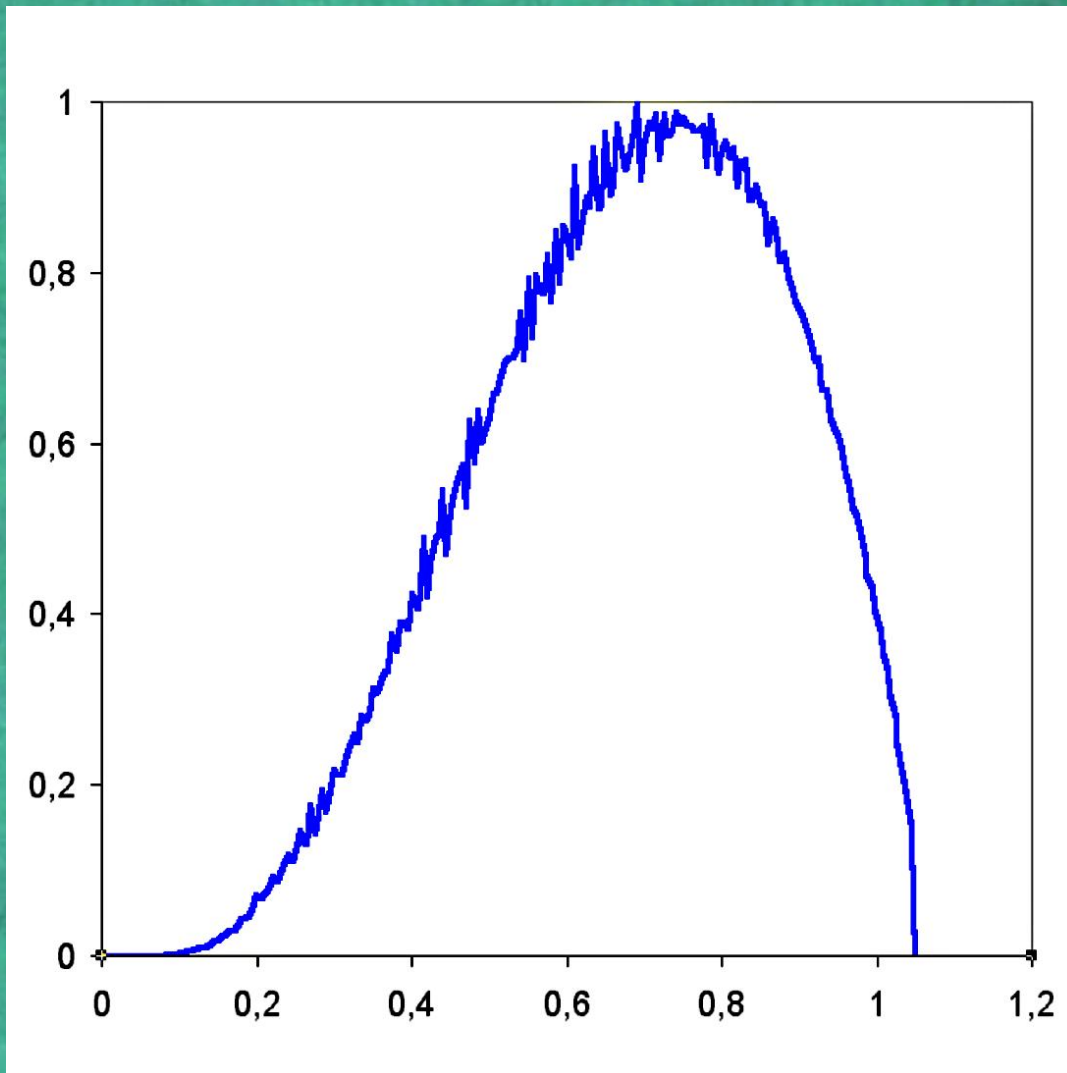
Keplerian

typical:

medium inclination,
standard emissivity,
2 relic Doppler peaks

Line classification

Bumpy



Parameters:

$$a = 0.998$$

$$i = 30^\circ$$

$$r_{\text{in}} = r_{\text{ms}} = 1.23$$

$$r_{\text{out}} = 30.0$$

$$\beta = 4.5$$

single power law

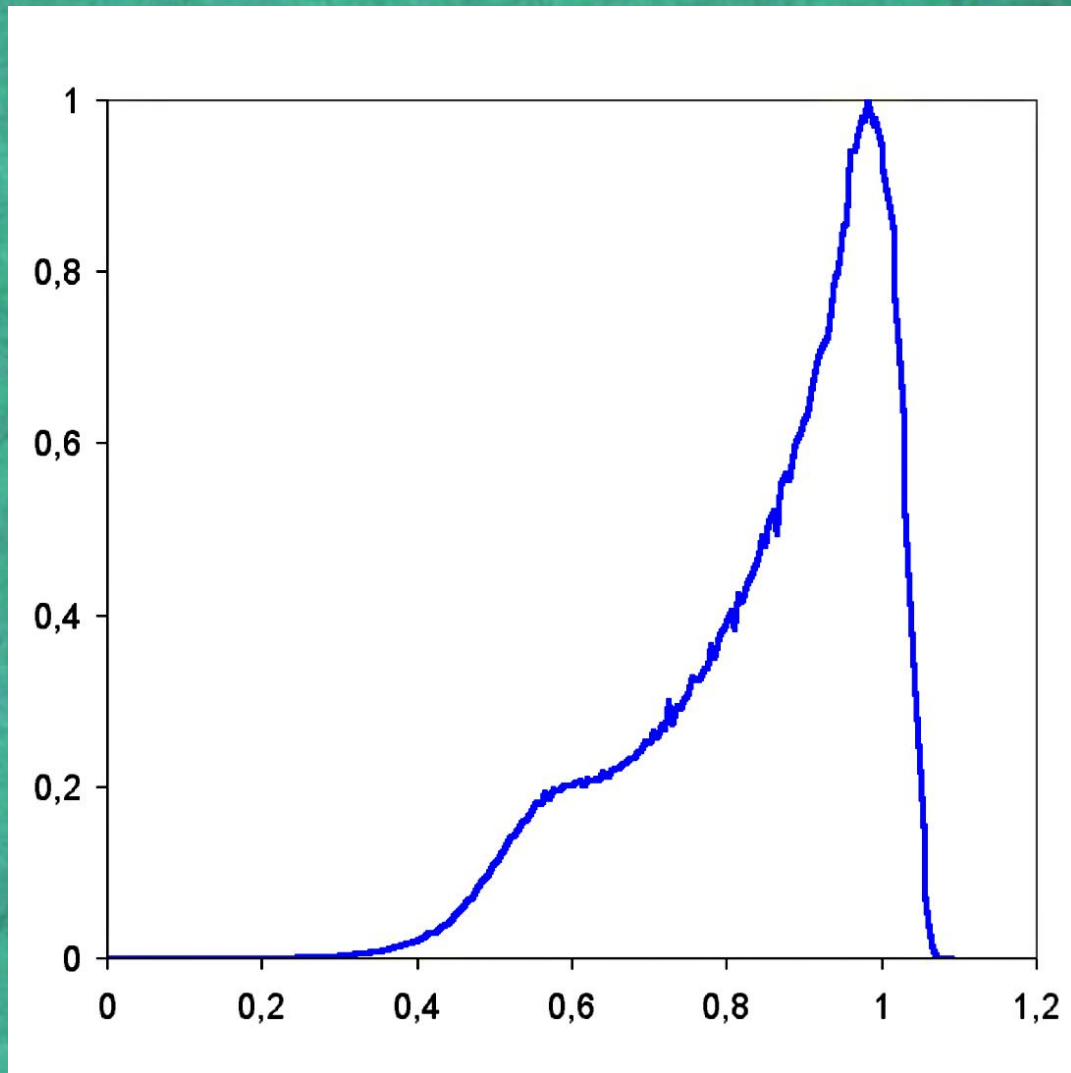
Keplerian

typical:

steep emissivity,
beaming lack

Line classification

Shoulder-like



Parameters:

$$a = 0.8$$

$$i = 40^\circ$$

$$r_{\text{in}} = 1.6$$

$$r_{\text{out}} = 30.0$$

$$R_t = 4.0$$

Gaussian emissivity

Keplerian + drift

typical:

localized emissivity,
Medium inclination,
very sensitive!

Line observations

Seyfert 1 MCG-6-30-15, $z = 0.008$

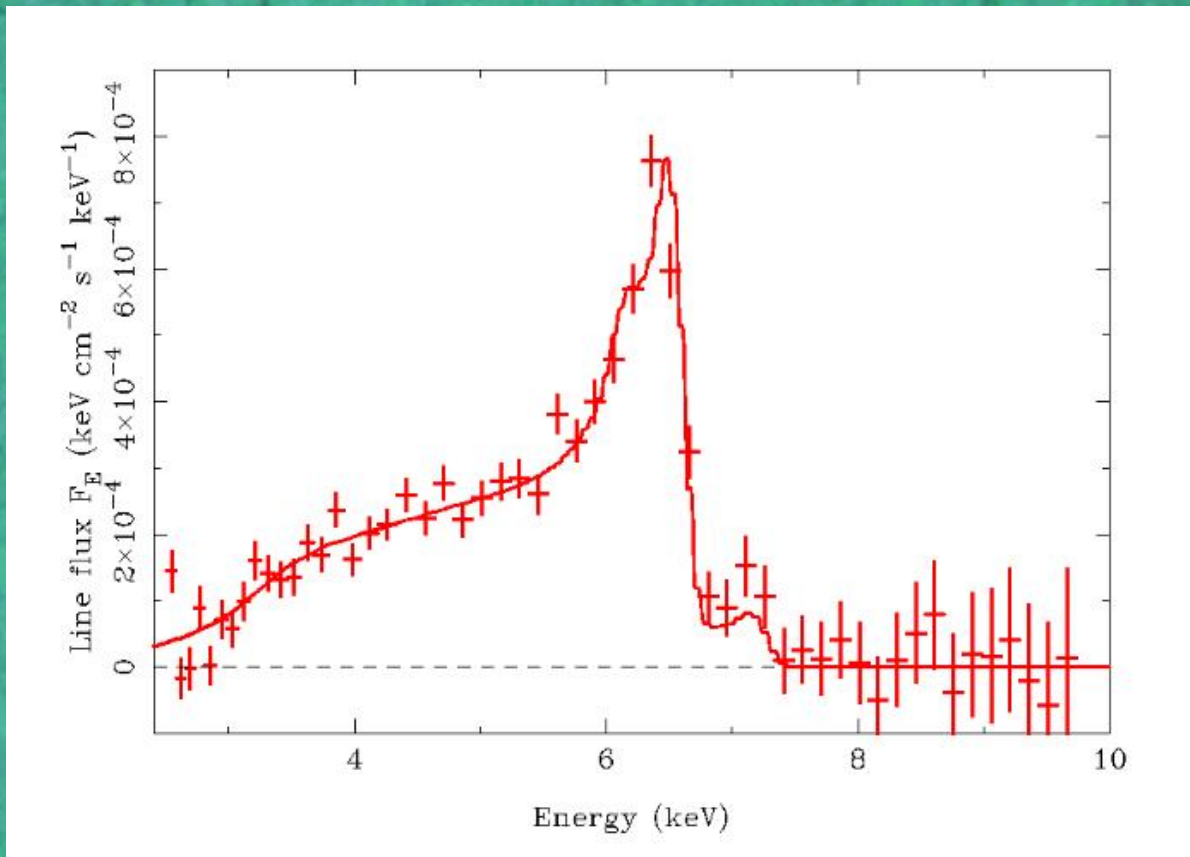
XMM EPIC MOS

broad Fe $K\alpha$ 6.5 keV
+
broad Fe $K\beta$ 7.05 keV

$i = 27.8^\circ$
 $R_{in} = 2.0$
 $R_{br} = 6.5$
 $q_{in} = 4.8$ broken
 $q_{out} = 2.5$ emissivity
 $\Gamma = 1.95$

shoulder-like
line topology

(Fabian et al. 2002)



Line observations

Seyfert 1.9 MCG-5-23-16, $z = 0.0083$

XMM EPIC PN

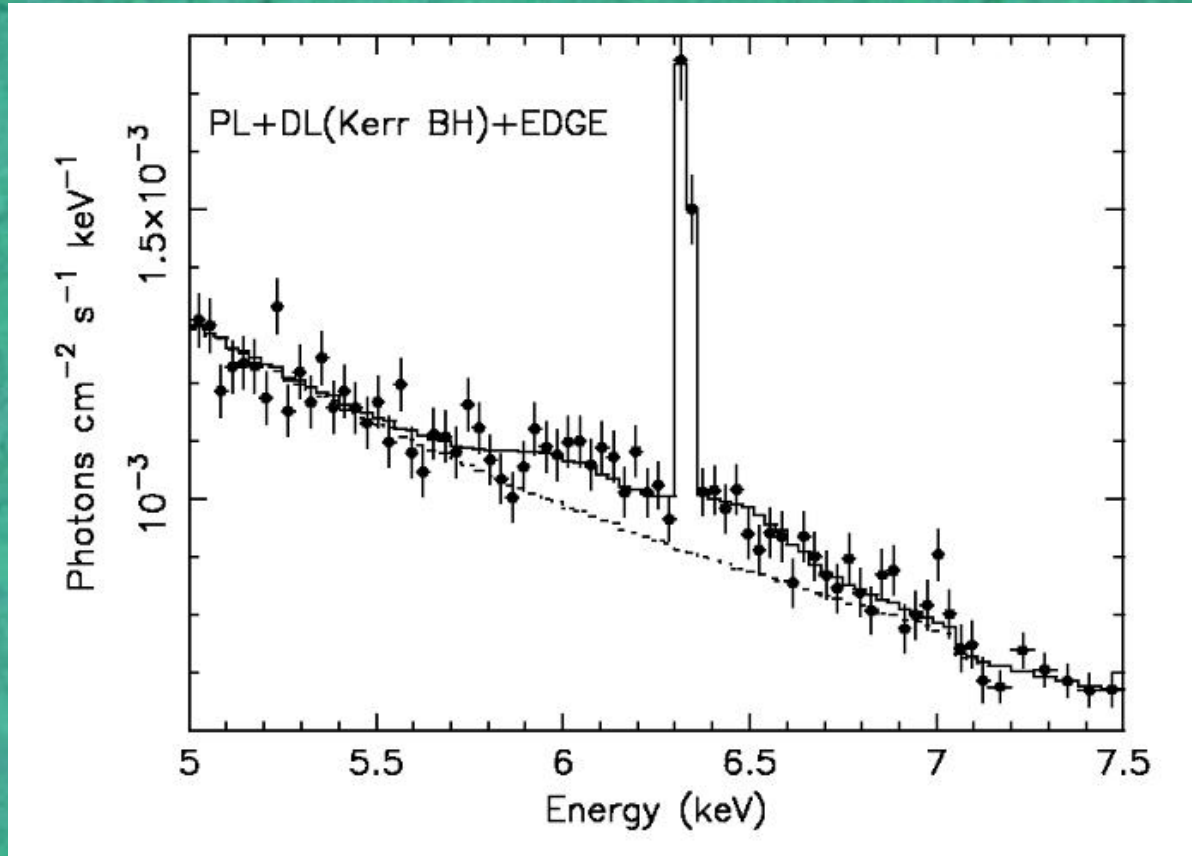
broad Fe $K\alpha$ 6.4 keV
+
narrow Gaussian
(torus reflection)

$i \sim 46^\circ$
absorption feature
at 7.1 keV

flattening continuum

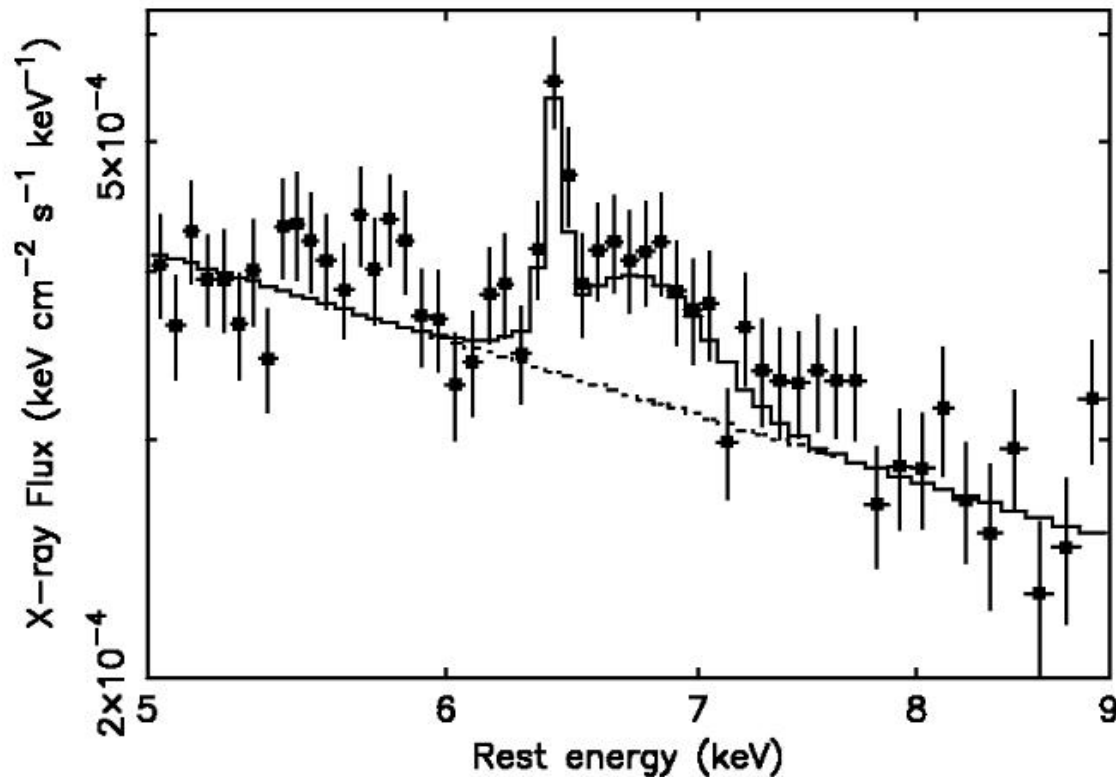
←→
line weakening

(Dewangan et al. 2003)



Line observations

Quasar Mrk 205, $z = 0.071$



XMM EPIC PN

broad Fe K α 6.7 keV
+
narrow Gaussian
6.4 keV (neutral
component)

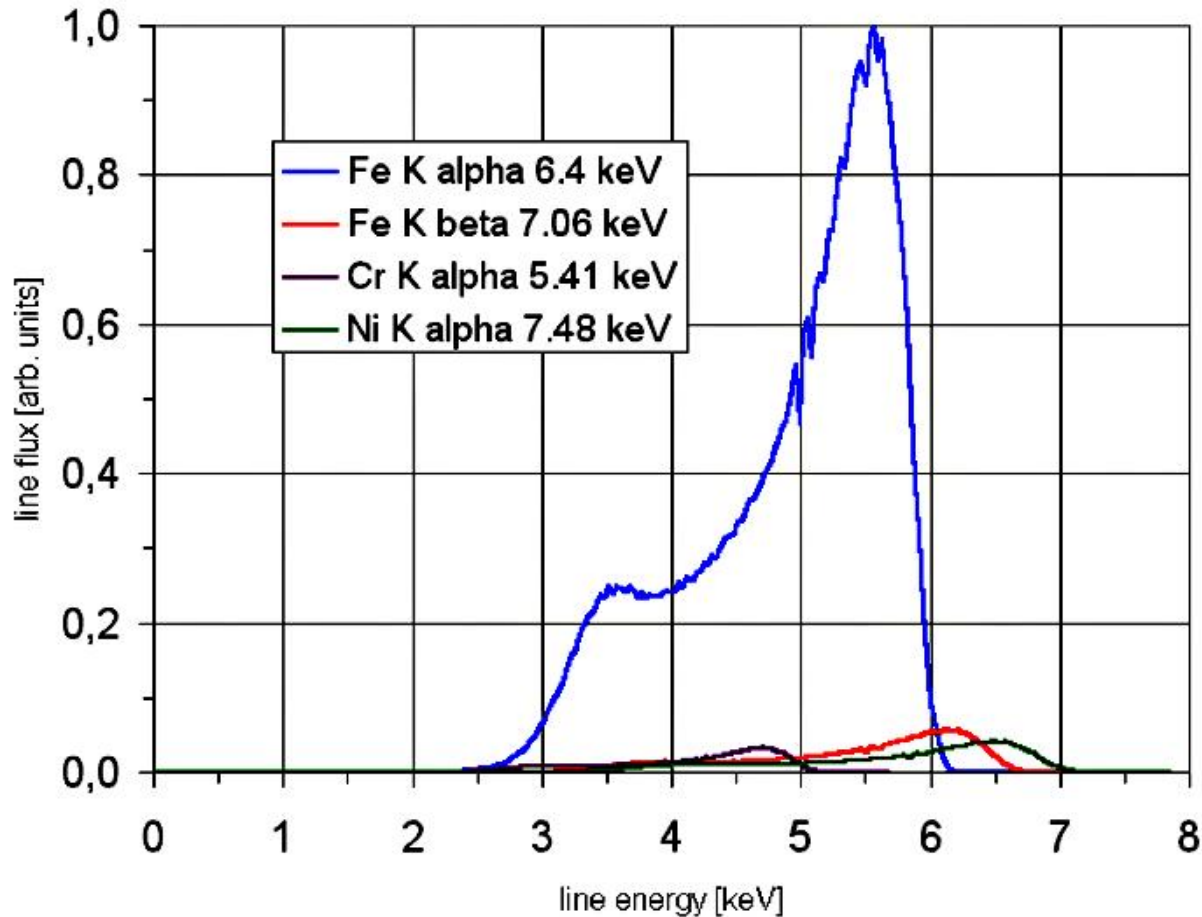
$i \sim 75 \dots 90^\circ$

low luminosity,
radio-quiet QSO

(Reeves et al. 2000)

X-ray spectroscopy

Multi-species emission line complex



Parameters:

$$a = 0.998$$

$$i = 30^\circ$$

$$r_{\text{in}} = r_{\text{ms}} = 1.23$$

$$r_{\text{out}} = 30.0$$

$$R_t = 4.0$$

$$\sigma_r = 0.8$$

Gaussian emissivity

(relative line strengths
from *Reynolds 1996*)

Coming soon on the web...

paper version of this talk

A. Müller & M. Camenzind (2003)

**powerpoint and postscript version of
this talk available under**

http://www.lsw.uni-heidelberg.de/~amueller/astro_ppt.html

References

- *Armitage & Reynolds 2003, astro-ph/0302271*
- *Balbus & Hawley 2002, ApJ, 573, 738 - 748*
- *Bromley et al. 1997, ApJ, 475, 57 - 64*
- *Carter 1968, Phys. Rev., 174, 1559*
- *Chandrasekhar 1983, The Mathematical Theory of Black Holes*
- *Cunningham 1975, ApJ, 202, 788 - 802*
- *Dewangan et al. 2003, astro-ph/0304037*
- *Fabian 1998, Astronomy & Geophysics, 123*
- *Fabian et al. 2002, MNRAS, 335, L1 – L5 (astro-ph/0206095)*
- *Falcke et al. 2000, ApJ, 528, L13*
- *Fanton et al. 1997, PASJ, 49, 159*
- *Hujeirat & Camenzind 2000, A&A, 361, L53 – L56*
- *Müller 2000, diploma*
- *Narayan & Yi 1994, ApJ, 428, L13*
- *Reeves et al. 2001, A&A, 365, L134 – L139 (astro-ph/0010490)*
- *Reynolds 1996, Ph.D.*
- *Reynolds & Nowak 2003, astro-ph/0212065*
- *Rybicki & Lightman 1979, Radiative Processes in Astrophysics*