Relativistic
Emission Lines
of
Accreting Black Holes

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Quasare und früher Kosmos
Forschungsseminar LSW
Overview

- Motivation
- AGN paradigm
- AGN X-ray spectra
- X-ray fluorescence
- Coronal irradiation
- Rotating black holes and Kerr ray tracing
- Plasma kinematics
- Accretion theory and radiation
- Radial drift model: disk truncation
- Simulated disk images: g-factor, emission
- Emission line calculation
- Emissivity models
- Emission lines: calculation, studies, criteria, classification, observation
Note:

All radius declaration were in units of the gravitational radius

\[ 1 \, r_g = \frac{GM}{c^2} \]

In general, relativistic units were used \( G=M=c=1 \)

\[ 1 \, r_g := 1.0 \]
Motivation

In general
- probing strong gravity
- verify or falsify event horizon?
  - Black Holes vs. Gravastars
    (few hope because strong redshift suppresses any information)
- measure parameters in accreting black hole systems
  (AGN, microquasars, globular clusters)

Cosmological
- emission line diagnostics for Quasars feasible
- highest redshift today: $z \sim 0.16$ (3C 273)
- extension to Early Universe expected
The AGN paradigm
Global topology: kpc-scale
X-ray emitter
Accreting black holes: pc-scale

AGN paradigm

Kerr black hole
hot toroidal corona

wind, jet

origin of Fe Kα emission line
cold thin truncated standard disk
X-ray AGN spectra
Spectral components

(plot idea by A. Fabian 1998)
X-ray fluorescence
Fe Kα
X-ray fluorescence
Prominent species

Fe K$_\alpha$ 6.40 keV
Fe K$_\beta$ 7.06 keV
Ni K$_\alpha$ 7.48 keV
Cr K$_\alpha$ 5.41 keV

dependency of these rest frame energies on ionization state!

(Reynolds 1996)
X-ray illumination
Corona geometries

- slab, sandwich
- sphere+disk geometry
- patchy, pill box

(Reynolds & Nowak 2003)
X-ray illumination

The corona problem

- corona geometry and location still open question!
- models:
  - slab corona (SSD, slim disk)
  - patchy corona
  - sphere+disk geometry (ADAF)
  - on-axis point-source (jet)
- observational technique: reverberation mapping
- theory: radiative GRMHD in 3D
Rotating Black Holes
Kerr geometry

\[ ds^2 = -\alpha^2 dt^2 + \tilde{\omega}^2 (d\Phi - \omega dt)^2 + \rho^2 / \Delta dr^2 + \rho^2 d\Theta^2 \]

Kerr metric in Boyer-Lindquist co-ordinates \( \{t, r, \Theta, \Phi\} \)

\[
g_{\mu\nu} = \begin{pmatrix}
g_{tt} & 0 & 0 & g_{t\Phi} \\
0 & g_{rr} & 0 & 0 \\
0 & 0 & g_{\Theta\Theta} & 0 \\
g_{\Phi t} & 0 & 0 & g_{\Phi\Phi}
\end{pmatrix} = \begin{pmatrix}
-\alpha^2 + \omega^2 \tilde{\omega}^2 & 0 & 0 & -\omega \tilde{\omega}^2 \\
0 & \rho^2 / \Delta & 0 & 0 \\
0 & 0 & \rho^2 & 0 \\
-\omega \tilde{\omega}^2 & 0 & 0 & \tilde{\omega}^2
\end{pmatrix}
\]

\[
g^{\mu\nu} = \begin{pmatrix}
g^{tt} & 0 & 0 & g^{t\Phi} \\
0 & g^{rr} & 0 & 0 \\
0 & 0 & g^{\Theta\Theta} & 0 \\
g^{\Phi t} & 0 & 0 & g^{\Phi\Phi}
\end{pmatrix} = \begin{pmatrix}
-1/\alpha^2 & 0 & 0 & -\omega / \alpha^2 \\
0 & \Delta / \rho^2 & 0 & 0 \\
0 & 0 & 1 / \rho^2 & 0 \\
-\omega / \alpha^2 & 0 & 0 & \frac{\alpha^2 - \omega^2 \tilde{\omega}^2}{\alpha^2 \tilde{\omega}^2}
\end{pmatrix}
\]

(Chandrasekhar 1983)
Numerical technique
Kerr ray tracing

curved space-time
black hole - disk system

ray

camera screen
observer

flat space-time

thin standard disk
Numerical technique
Geodesics equations in Kerr

- GR Lagrangian in Boyer-Lindquist co-ordinates
- Legendre transformation to Hamiltonian
- separability ansatz for Hamilton-Jacobi differential equation
- photon momenta follow from derivatives of action
- 4 conservatives:
  - energy $E$
  - mass $\mu$
  - angular momentum $J$
  - Carter constant $C$ (Kerr-specific!)
- reduction to set of 4 1st order differential equations
- integration of geodesics equations by
  - Runge-Kutta scheme (direct method)
  - elliptical integrals (Fanton et al. 1997, A. Müller 2000)
  - transfer functions (Cunningham 1975, Bromley et al. 1997)

(Chandrasekhar 1983)
Generalized Doppler factor

**g-factor**

\[
g \equiv \frac{\nu_{\text{obs}}}{\nu_{\text{em}}} = \frac{\hat{p}_t^{\text{obs}}}{\hat{p}_t^{\text{em}}}
\]

**definition in rest-frame**

Carter momenta in ZAMO (1968)

\[
\hat{p}_t = \gamma \left[ p(t) - v(j)p(j) \right]
\]

Lorentz boost from ZAMO to rest frame

\[
g = \frac{\alpha}{\gamma \left[ (1 - \omega \lambda) - \alpha v^{(r)} \frac{\sqrt{R_0}}{\rho \sqrt{\Delta}} - \alpha v^{(\theta)} \frac{\sqrt{\Theta}}{\rho} - \alpha v^{(\Phi)} \frac{\lambda}{\omega} \right]}
\]

\[
= \frac{\alpha}{\gamma \left[ 1 - \alpha v^{(r)} \frac{\sqrt{R_0}}{\rho \sqrt{\Delta}} - \alpha v^{(\theta)} \frac{\sqrt{\Theta}}{\rho} - \lambda \Omega \right]}
\]
Plasma kinematics

**Keplerian**
- \( v_\phi = v_{\text{Kepler}} \)
- \( v_r = 0 \)
- \( v_\theta = 0 \)

**non-Keplerian**
- \( v_\phi = v_{\text{Kepler}} \)
- \( v_r \neq 0 \)
- \( v_\theta = 0 \)

**non-Keplerian**
- \( v_\phi \neq 0 \)
- \( v_r \neq 0 \)
- \( v_\theta \neq 0 \)

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**classical approach**

**radial drift model**

**radiative 3D-GRMHD**
Accretion theory
Hydrodynamics and MHD

- co-existent and overlapping solutions available:
  - ADAF (Advection-Dominated Accretion Flow)
    Narayan & Yi 1994
  - ADIOS (Advection-Dominated Inflow-Outflow Solution)
    Blandford & Begelman 1999
  - CDAF (Convection-Dominated Accretion Flow)
    Quataert & Gruzinov 2000
  - ISAF (Ion-Supported Accretion Flow)
    Spruit & Deufel 2001
  - TDAT (Truncated Disk – Advective Tori)
    Hujeirat & Camenzind 2001
  - NRAF (Non-Radiative Accretion Flow)
    Balbus & Hawley 2002

- $\alpha$- and $\beta$-disks
- complete parameter space investigation
- need for covariant radiative generalization!
Radiation mechanisms

- **thermal emission**
  - single black body
  - multi-color black body (SSD)
- **Comptonization** (Kompaneets equation)
  - dominant global X-ray component
  - reprocessed soft photons from environment
  - corona: seed photon production for fluorescence
- **Synchrotron radiation**
  - radio emission
  - fast cooling of hot accretion flow on ms-scale
  - SSC (sub-mm bump)
  - SSA (dip feature)
- **bremsstrahlung**
  - launch of outflow (disk wind, Poynting flux)

→ Covariant generalization: GR radiation transfer!
Truncated Standard accretion Disks (TSD) due to efficient radiative cooling. Disk cuts off at $R_t$, not at $r_{ms}$ (cp. SSD) depending on radiative accretion theory (accretion rate, cooling, conduction).

(Hujeirat & Camenzind 2000)
Radial drift model
Velocity field in ZAMO frame

ZAMO velocities

\[ v^{(\Phi)} = \tilde{\omega} \left( \frac{\Omega - \omega}{\alpha} \right) \]

\[ v^{(r)} = \frac{\sqrt{R}}{\Sigma (1 - \omega \lambda)} \]

Angular frequencies

\[ \Omega_K = \pm \frac{\sqrt{M}}{\sqrt{r^3} \pm a \sqrt{M}} \]

\[ \Omega = \Omega_{in} = \omega + \frac{\alpha^2}{\tilde{\omega}^2} \frac{\lambda_{ms}}{1 - \omega \lambda_{ms}} \]

\[ \lambda_{ms} = \frac{\tilde{\omega}_{ms}^2 (\Omega_{K,ms} - \omega_{ms})}{\alpha_{ms}^2 + \omega_{ms} \tilde{\omega}_{ms}^2 (\Omega_{K,ms} - \omega_{ms})} \]
Radial drift model
Parameter restrictions

\[ \lambda = \frac{J}{E} \]

Specific angular momentum \( \lambda \) has chosen between \( \lambda_{ms} \) and \( \lambda_{mb} \).

Only region between \( \Omega_+ \) and \( \Omega_- \) is allowed (time-like trajectories).
Radial drift models

Radial ZAMO velocity vs. radius \([r_g]\)
Rendered disk images

g-factor and emission

\[ a = 0.8 \]
\[ i = 80^\circ \]
\[ r_{in} = r_H = 1.6 \]
\[ r_{out} = 4.6 \]
\[ R_t = 3.0 \]
\[ \sigma_r = 3.0 \]
Disk emission
Relativistic effects

Equatorial emitting ring with orbiting free-falling matter at high inclination

- lensed disk segment
- event horizon
- rotating black hole (Kerr solution)
- front beaming (enhanced emission)
- back beaming (reduced emission)
- gravitational redshift
Radial drift model

g-factor: Keplerian vs. Drift

pure Keplerian  Keplerian plus radial drift

\[ a = 0.1 \]
\[ i = 40^\circ \]
\[ r_{in} = r_H = 1.996 \]
\[ r_{out} = 10.0 \]
\[ R_t = 5.0 \]
Radial drift model
Implications

- adequate consideration of **accreted inflow**
- **truncation** softens the „evidence for Kerr“- argument, because $R_t$ replaces $r_{ms}$. Coupling between $r_{in}$ and $r_{ms}$ is lost!
- **gravitational redshift** is enhanced!
- emission line shape does not change dramatically compared with pure Keplerian: only **red wing effects**
- **poloidal motion** still neglected!
- awaiting new accretion theory: **covariance**
- follow *Armitage & Reynolds (2003)* approach: couple line emission to accretion model
Disk emission
Inclination study with $g^4$

The Shadow of Kerr Black Holes

01° 05° 10° 15° 20°
25° 30° 35° 40° 45°
50° 55° 60° 65° 70°
Strong gravitational redshift

horizon:
g = 0

Flux integral folds $g$ in high power with emissivity.

$g^4$ – distribution suppresses *any emission* near black holes!

„Shadow“
by Falcke et al. 2000
Relativistic emission line Calculation

\[ F^{\text{obs}} = \int_{\text{image}} d\Omega \ I^{\text{obs}}_\nu \]

\[ I^{\text{obs}}_\nu = g^3 \ I^{\text{em}}_\nu \]

\[ \hat{F}^{\text{em}}_\nu = \pi \hat{I}^{\text{em}}_\nu = \epsilon(r) \delta (\nu_{\text{em}} - \nu_0) \]

\[ F_{\text{obs}}(E_{\text{obs}}) = \int_{\text{image}} \epsilon(r) g^4 \delta(E_{\text{obs}} - gE_0) d\Omega \]

general spectral flux integral

using Lorentz invariant
\((\text{Misner 1973})\)

assume line shape in rest frame:

\[ \delta\text{-distribution} \]

fold radial emissivity profile

- single power law
- double or broken power law
- Gaussian
- cut-power law

evaluate tuple \(\{g, \Delta\Omega, r\}\) on each pixel and sum over pixels!
Radial emissivity profiles

**Emissivities**

- Single power law: \( r^{-\beta} \)
  - Gaussian: \( \exp\left[-\frac{(r - R_t)^2}{\sigma^2_r}\right] \)
  - Cut power law: \( \exp\left[-\frac{R_t}{r}\alpha\right]r^{-\beta} \)

- Double or broken power law
  (Page & Thorne 1974)

- Gaussian, cut-power law
  (Müller & Camenzind 2003)
Line features
Imprints of relativistic effects

- Doppler (Newtonian)
- Beaming (SR)
- Gravitational redshift (GR)
Line studies
Inclination

Parameters:
\[ a = 0.999999 \]
\[ i = 5^\circ \ldots 70^\circ \]
\[ r_{in} = r_{ms} = 1.0015 \]
\[ r_{out} = 30.0 \]

single power law
emissivity

pure rotation,
no drift

Blue edge shifts!
Enhanced Beaming!
Doppler effect
Line studies
Inner disk edge

Parameters:
\( a = 0.999999 \)
\( i = 30° \)
\( r_{\text{in}} = 1...28 \)
\( r_{\text{out}} = 30.0 \)

single power law
emissivity
pure rotation,
no drift

Static blue edge!
Red wing vanishes!
Doppler effect
end: Newtonian

Space-time curvature
is negligible at
radii \( \sim 20 \, r_g \)!!!
Line studies
Outer disk edge

Parameters:
\[ a = 0.999999 \]
\[ i = 30° \]
\[ r_{\text{in}} = 1.0015 \]
\[ r_{\text{out}} = 30 \ldots 1.5 \]

single power law emissivity

pure rotation, no drift

Static red edge!
Beaming vanishes!
Doppler effect
Line studies
Kerr parameter

Parameters:
- \( a = 0.1 \ldots 0.999999 \)
- \( i = 40^\circ \)
- \( r_{in} = r_{ms} \)
- \( r_{out} \sim 10.0 \) decreasing

constant emitting area!

single power law emissivity

pure rotation, no drift

Beaming increases due to increasing frame-dragging effect!
Line studies
Truncation radius

Parameters:
- $a = 0.1$
- $i = 40^\circ$
- $r_{in} = r_H = 1.995$
- $r_{out} = 30.0$
- $R_t = 4 \ldots 8$
- $\sigma_r = 0.4 \, R_t$

Gaussian emissivity couples to $R_t$

non-Keplerian:
rotation plus drift!

Gravitational redshift decreases with radius!
Enhanced Beaming!
Doppler effect
Line studies
Drift + rotation vs. pure rotation

Parameters:
\( a = 0.001 \)
\( i = 30^\circ \)
\( r_{in} = r_H = 2.0 \)
\( r_{out} = 30.0 \)
\( R_t = 6 \)

single power law emissivity

pure Keplerian
non-Keplerian: rotation plus drift!

Drift causes enhanced gravitational redshift and reduces red wing flux!
Line studies
Drift + rotation vs. pure rotation

Parameters:
\( a = 0.1 \)
\( i = 40^\circ \)
\( r_{\text{in}} = r_{H} = 1.995 \)
\( r_{\text{out}} = 10.0 \)
\( R_t = 5 \)
\( \sigma_r = 0.4 \, R_t \)

Gaussian emissivity couples to \( R_t \)

pure Keplerian
non-Keplerian: rotation plus drift!

Gravitational redshift causes red wing differences!
Line suppression
„Shadowed lines“

Parameters:

- $a = 0.998$
- $i = 30^{\circ}$
- $r_{in} = r_{H} = 1.06$
- $r_{out} = 30.0$
- $R_{t} = 1.5$
- $\sigma_{r} = 0.4$

Gaussian emissivity non-Keplerian:
rotation + drift
peak at $\sim 3$ keV
high redshift!

(„unphysical“ line: consider fluorescence restrictions)
Line criteria

**DPR**
Doppler Peak Ratio

**DPS**
Doppler Peak Spacing

(relative quantities!)

(Müller & Camenzind 2003)
Line classification
Proposed nomenclature

- topological criterion:
  - triangular
  - bumpy
  - double-horned
  - double-peaked
  - shoulder-like

- pre-selection of parameters possible
- pre-classification of observed lines
- unification scheme of AGN
Line classification
Triangular

Parameters:

\[ a = 0.999999 \]
\[ i = 10^\circ \]
\[ r_{\text{in}} = 1.0015 \]
\[ r_{\text{out}} = 30.0 \]
\[ \beta = 3.0 \]

single power law

Keplerian

*typical:*
low inclination, Doppler reduced
Line classification
Double-peaked

Parameters:
\( a = 0.999999 \)
\( i = 30^\circ \)
\( r_{\text{in}} = 28.0 \)
\( r_{\text{out}} = 30.0 \)
\( \beta = 3.0 \)

single power law

Keplerian

typical:
medium2high inclination, asymptotically
flat metric, no GR effects
Line classification
Double-horned

Parameters:
\( a = 0.4 \)
\( i = 40^\circ \)
\( r_{\text{in}} = 1.9165 \)
\( r_{\text{out}} = 9.9846 \)
\( \beta = 3.0 \)

Single power law
Keplerian

Typical:
medium inclination,
standard emissivity,
2 relic Doppler peaks
Line classification
Bumpy

Parameters:

- $a = 0.998$
- $i = 30°$
- $r_{\text{in}} = r_{\text{ms}} = 1.23$
- $r_{\text{out}} = 30.0$
- $\beta = 4.5$

Single power law

Keplerian

typical:
steep emissivity,
beaming lack
Line classification
Shoulder-like

Parameters:
\( a = 0.8 \)
\( i = 40^\circ \)
\( r_{\text{in}} = 1.6 \)
\( r_{\text{out}} = 30.0 \)
\( R_{t} = 4.0 \)

Gaussian emissivity
Keplerian + drift

*typical:*
localized emissivity,
Medium inclination,
very sensitive!
Line observations
Seyfert 1 MCG-6-30-15, z = 0.008

XMM EPIC MOS

broad Fe Kα 6.5 keV
+ broad Fe Kβ 7.05 keV

i = 27.8°
R_{in} = 2.0
R_{br} = 6.5
q_{in} = 4.8 broken
q_{out} = 2.5 emissivity
Γ = 1.95

shoulder-like
line topology

(Fabian et al. 2002)
Line observations
Seyfert 1.9 MCG-5-23-16, z = 0.0083

XMM EPIC PN
broad Fe Kα 6.4 keV +
narrow Gaussian (torus reflection)
i ~ 46°
absorption feature at 7.1 keV
flattening continuum
line weakening

(Dewangan et al. 2003)
Line observations
Quasar Mrk 205, $z = 0.071$

**XMM EPIC PN**

broad Fe K$\alpha$ 6.7 keV
+
narrow Gaussian
6.4 keV (neutral component)

$i \sim 75...90^\circ$

low luminosity,
radio-quiet QSO

*(Reeves et al. 2000)*
X-ray spectroscopy
Multi-species emission line complex

Parameters:
\[ a = 0.998 \]
\[ i = 30° \]
\[ r_{in} = r_{ms} = 1.23 \]
\[ r_{out} = 30.0 \]
\[ R_t = 4.0 \]
\[ \sigma_r = 0.8 \]
Gaussian emissivity

(relative line strengths from Reynolds 1996)
Coming soon on the web...

paper version of this talk
A. Müller & M. Camenzind (2003)

powerpoint and postscript version of this talk available under
http://www.lsw.uni-heidelberg.de/~amueller/astro_ppt.html
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