An introduction to brane world cosmology

Andreas Müller
Theory group LSW

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http://www.lsw.uni-heidelberg.de/users/amueller
Overview

- principles
- bulk and brane
- extradimensions
- compactification
- ADD vs. Randall-Sundrum
- scalar fields
- brane collisions
- ekpyrosis and cyclic universe
Standard cosmology

- GR world is 4D manifold: space-time
- Robertson-Walker metric
- $\Lambda$ cosmology
- cosmological constant, dark energy
- $\Lambda$CDM in a flat, expanding universe
- FRW equations
- Hubble constant
- inflation
- Big Bang
Motivation to brane world

- coincidence problem:
  $\Omega_\Lambda \sim \Omega_m$
  solution: $\Lambda$ becomes dynamical
  quintessence models (QCDM),
  brane worlds

- hierarchy problem:
  weakness of gravition!
  Planck scale $\sim 10^{19}$ GeV
  electroweak scale $\sim 1$ TeV
  16 decades discrepancy!
  solution: extradimensions,
  brane worlds
Extradimensions and brane worlds

- extradimensions (XDs):
  ~1920: Nordström, Kaluza-Klein
  ~1990: renaissance in QFT, SUSY; Antoniadis

- implications from string theories and M-theory: compactified extradimensions

- count XDs in particle accelerator black holes?

- standard model of particle physics is confined on a hypersurface, the **brane**
  (etymology from *membrane* by Paul Townsend: *p*-brane has dimension *p*)

- brane is embedded in higher-dimensional space, the **bulk**
Bulk – brane topology

brane
standard model fields

bulk
graviton
Brane world zoo

- number of extradimensions
- compactification vs. non-compactification
- flat vs. warped bulk geometry
- number of branes
- static vs. dynamical branes (brane collisions)
- vacuum bulk vs. bulk scalar fields
Supersymmetry

SUSY mirror creates particle zoo
String theory

- 5 supersymmetric string theories connected via dualities hint for M-theory
- 11D supergravity (SUGRA) connects GR with SUSY
- SUGRA is low-energy limits (l >> l_pl) of M-theory and therefore all string theories
- 11D SUGRA has 11th dimension compactified on an orbifold (with Z_2 symmetry)
- boundaries of 11D space-time are 10D „planes“
- on planes E_8 gauge groups confined
- Calabi-Yau threefold represents compactified space of 6 dimensions of 11D („microscopic ball“)
- heterotic string theory E_8 x E_8 results in brane world (Horava & Witten 1996)
String theory: ADD model

- motivation for 5D space-times with 4D boundary branes
- ADD scenario: **large extradimensions (LXD)**s
- flat bulk geometry 4+d
- d compactified extradimensions
- reduced Planck scale:
  \[ M_{P,ADD}^2 = M_{fund}^{2+d} R^d \]
  \( M_{fund} \): 4+d Planck scale
- radii < R: **non-Newtonian gravity**
Newton’s law modified

- SM restricted to brane, gravity propagates into bulk
- extradimensions compactified to radius $R$
- $1^{\text{st}}$ implication: **Newton $1/r^2$ injured** for radii $\sim R$
- tests with Cavendish experiments show **no evidence** up to now
- if LXD exist, then $R \ll 1 \text{ mm}$
2-brane system

hypersurface: $D_{brane} = D_{bulk} - 1$
Randall-Sundrum I model

- 2-brane system
- warped (curved) bulk geometry 4+d
- bulk metric is slice of **Anti de Sitter** (AdS$_5$) space-time, $\Lambda < 0$, 5D:
  \[
  ds^2 = e^{-2K(y)} \eta_{\mu\nu} \, dx^\mu \, dx^\nu + dy^2
  \]

- **new:** restoration of Newton‘s law on brane with positive tension embedded in infinite LXD!
- solution of the **hierarchy problem**
  (10$^{19}$ GeV Planck vs. 100 GeV electroweak):
  **2-brane model (RSI)**
Randall-Sundrum I model

Remark: branes are Minkowski-flat
Randall-Sundrum I model

- highly-curved AdS background
  - implies large gravitational redshift of energy-scale between branes
- hierarchy due to large inter-brane distance $r_c$
- Planck scale (on negative tension brane) is reduced to TeV:
  $$M^2_{P,RS} \sim \exp(2kr_c) M_5^3/k, \ k = (-\Lambda_5/\kappa_5^2)^{1/2}$$
  $\Lambda_5$: 5D negative cosmological constant on bulk
  $\kappa_5$: 5D gravitational coupling constant
  $M_5$: 5D Planck mass
- fine tuning problem:
  radius of LXD, $r_c$, tunes hierarchy scale
- radion as bulk scalar field (later!)
Randall-Sundrum II model

- AdS background
- send negative tension brane to infinity
- effectively **non-compact** 1-brane model
- contrast to KK (all XDs compactified):
  gravitational field has **continuum** of KK modes
- consequence:
  correction of gravitational force on brane
Randall-Sundrum II model

- modified Newton potential for point masses on the brane

$$V(r) = \frac{G_N m_1 m_2}{r} \left( 1 + \frac{l^2}{r^2} + O(r^{-3}) \right)$$

with $l^2 = -6/(\Lambda_5 \kappa_5^2)$

- experiments prove $l < 1$ mm
Randall-Sundrum II model

- modified Friedmann equation in 5D

\[ H^2 = \frac{8\pi G}{3} \rho_M \left[ 1 + \frac{\rho_M}{2\sigma} \right] + \frac{\Lambda_4}{3} + \frac{\mu}{a^4} \]

\[ \frac{8\pi G}{3} = \frac{\sigma}{18} \]

\[ \frac{\Lambda_4}{3} = \frac{\sigma^2}{36} + \frac{\Lambda_5}{6} \]

- tuning between \( \Lambda_5 \) and \( \sigma \) establishes \( \Lambda_4 = 0 \)
- gravitational constant depends on tension \( \sigma \)
- \( \mu \) is dark radiation term
Observational constraints

- **nucleosynthesis**
  \[ \sigma > (1 \text{ MeV})^4, \]
  then classical Friedmann eq. established at \( z_{\text{nucl}} \), otherwise abundances significantly changed

- **Newton’s law tests**
  \[ \sigma > (100 \text{ GeV})^4, \kappa_{-3} > 10^5 \text{ TeV}, \]
  then classical Friedmann eq. established at \( z_{\text{nucl}} \), otherwise abundances significantly changed

- **cosmology**
  \[ \mu < 0.1 \rho_{\text{phot}}; \text{ typically assumed } \mu = 0 \]
Technical aspects

- start with action (Einstein-Hilbert, ansatz for brane: contains tension)
- derive **Einstein equations** as EOM, including **Klein-Gordon equation**
- solve this set of equations (integration...)
- deduce **bulk metric** (AdS, Schwarzschild etc.)
- identify **tunings** ($\Lambda_5 - \sigma$ – relation etc.)
- discuss resulting **cosmology**, e.g. modified Friedmann equations, effective cosmological constants...
Bulk scalar field

brane 1
(universe 1)

bulk
represented by scalar field
(radion, cosmon)

brane 2
(universe 2)
Bulk scalar field

- up2now: empty bulks
- now: fill bulk with scalar field
- dynamical brane configurations!
- bulk back reaction parametrized by Weyl tensor and loss parameter
- discuss modified Friedmann eq.
- Klein-Gordon eq.:
  - time dependence of scalar field
    - trace of energy-stress tensor on brane
    - gradient of bulk potential
- G becomes time-dependent: $G = G(z)$
- fine-structure constant has time evolution
- bulk scalar field can play role of quintessence
Scalar field

- energy density, pressure, potential energy

\[
\rho_\phi = \frac{1}{2} \phi_{\mu} \phi^{\mu} + V(\phi), \\
p_\phi = \frac{1}{2} \phi_{\mu} \phi^{\mu} - V(\phi)
\]

- full evolution described by:
  - modified Friedmann eq.
  - Klein-Gordon eq.
  - Raychaudhuri eq.

- assume *slow-roll* regime
- **result**: brane world effects slow-roll scenarios
Scalar field - inflaton

- in slow-roll regime (1):
  high potential vs. low kinetic energy of scalar field
- high negative pressure drives expansion of universe
- fall into potential well (2):
  inflation ends, inflaton field oscillates and decays into matter and radiation

\[
\rho_\phi = \frac{1}{2} \phi_{\mu\nu} \phi^{\mu\nu} + V(\phi)
\]
\[
p_\phi = \frac{1}{2} \phi_{\mu\nu} \phi^{\mu\nu} - V(\phi)
\]

figure: Steinhardt & Turok 2002
Cosmology of 2-brane systems

- motivation: 1-brane system + scalar field generates **naked singularity** (bulk singularity, AdS horizon). This can be shielded with 2\textsuperscript{nd} brane.
- bulk scalar field **fixes** inter-brane distance in RSI model
- consider **variable** inter-brane distance
- **radion**: inter-brane distance plays role of scalar field
- small radion field at late times: negative tension brane moves towards bulk singularity and might be destroyed or repelled
Cosmological constant

- observed $\Lambda \sim 0$ invokes extradimension effect
- hierarchy problem reemerges in a fine tuning problem of the inter-brane distance
- **Friedmann equations modified** at high energies ($\rho_m >> \sigma$) in brane world models:

  \[
  H \sim \rho_m
  \]

  instead of classical 4D:

  \[
  H \sim \rho_m^{1/2}
  \]
Ekpyrotic scenario

- initial state two flat 3-branes: our progenitor universe and a „parallel“ universe
- branes approach as mediated by radion field
- in *brane collision* event kinetic energy is transformed into quarks and leptons
- **no big bang singularity!**
- finite temperature $10^{23}$ K
- homogeneous and flat universe
- **no** inflation!
- no magnetic monopole formation (T too small)

*Khoury et al. 2001*
Cyclic Universe

- periodic sequences of ekpyrosis
- cycle of big bang, expansion, contraction, big crunch
- scalar field acts as dark energy (precisely quintessence) that accelerates and decelerates
- scalar field has natural geometrical interpretation in string theory

Steinhardt & Turok 2001
Cyclic Universe

- (1) $E_{\text{pot}}$ dominant
- (2) roll to well due to universe expansion and cooling
- (3) $E_{\text{pot}} = 0$, $E_{\text{kin}}$ dominates universe, expansion decelerates
- (4) $E_{\text{pot}} < 0$, contraction
- (5) acceleration out of the minimum, scale factor zero: „crunch“
- (6) reheating of universe from kinetic energy conversion into matter and radiation
- (7) rush back

Steinhardt & Turok 2002
Brane Worlds – συν–οΨις

- existence of extradimensions
- $\Lambda = 0$ on the brane easily managed
- impact of brane cosmology on early universe
  - $H \sim \rho_m$ instead of $H \sim \rho_m^{1/2}$
- dark energy, quintessence represented by scalar field
- ekpyrosis: 1st explanation of big bang!
- universe may evolve in cycles
Open questions

- effects of bulk gravitation on CMB and LSS
- boundary conditions on the brane
- variations of the bulk scalar field around the brane
- bulk scalar field as dark energy constituent
- shielded bulk singularity
- singularity problem in brane collisions
Cosmology news

- $w = p/\rho = -1$  *Einstein's cosmological constant* $\Lambda$
  high-z SN Typ Ia permanence measurements
  *(Riess et al., February 2004)*

- distance ladder
  $z \sim 7$ lensed IR galaxy
  *(Kneib et al., February 2004)*

**z \sim 10** lensed IR galaxy Abell 1835 IR 1916
  lens magnification factor 25-100, $5 \times 10^8$ M$_\odot$, ISAAC/VLT
References

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**Abbreviations and Acronyms**

- **ADD**: Arkani-Hamed, Dimopoulos & Dvali model
- **AdS**: Anti de Sitter space-time
- **BH**: Black Hole
- **CMB**: Cosmic Microwave Background
- **D**: Dimension
- **EOM**: Equation of Motion
- **FRW**: Friedmann-Robertson-Walker
- **GR**: General Relativity
- **GW**: Gravitational Wave
- **KGE**: Klein-Gordon Equation
- **KK**: Kaluza-Klein
- **ΛCDM**: Λ cosmology with cold dark matter
- **LSS**: Large Scale Structure
- **LXD**: Large Extra Dimension
- **QCDM**: quintessence cosmology with cold dark matter
- **QFT**: Quantum Field Theory
- **RSI**: Randall-Sundrum model I
- **RSII**: Randall-Sundrum model II
- **SM**: Standard Model of Particle Physics
- **SUGRA**: supergravitation
- **SUSY**: supersymmetry
- **XD**: Extra Dimension